Port-thermodynamic systems' control

B. Maschke and A.J. van der Schaft

LAGEPP, UMR 5007 CNRS-Université Lyon 1, France and
Jan C. Willems Center for Systems and Control, University of Groningen, The Netherlands

Joint Structures and Common Foundation of Statistical Physics, Information Geometry and Inference for Learning, Les Houches, 26th July to 31st July 2020

Introduction and motivation

Introduction and motivation

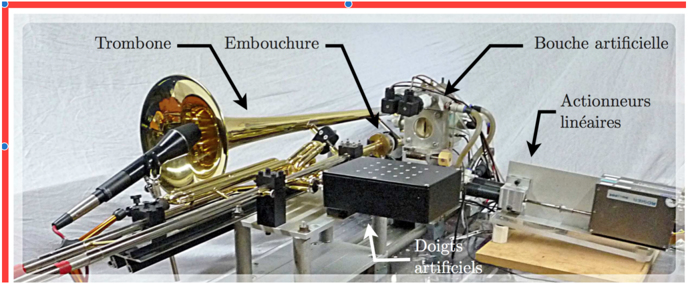
Context and motivation

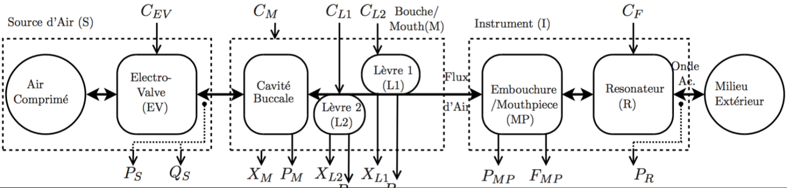
Use physical invariants and couplings in the :

1. physically-based modelling making use of physical invariants and port (conjugated interface) variables
2. physically-based simulation making use of physical invariants
3. physically-based control design : design control Lyapunov functions using physical invariants
4. simultaneous design and control using *physical analogy* of the controller or the closed-loop system

In this talk we use Hamiltonian control systems for these objectives !

Port Hamiltonian systems for a robotic system playing trombone [N. Lopes, IRCAM, 2016].





Structure-preserving control of dissipative Hamiltonian systems

- Assigning the Hamiltonian function for *input-output Hamiltonian systems* on symplectic manifolds.
  [van der Schaft, in Theory and Applications of Nonlinear Control Systems, 1986]
- Assigning the structure matrices, Hamiltonian of *port Hamiltonian systems*
  [R. Ortega et al., **IEEE Control Systems Magazine**, 2001]

Structure preserving control of controlled Hamiltonian systems

For Hamiltonian control systems defined on symplectic manifolds T^*Q where Q is the configuration space :

$$\dot{x} = X_{H_0} - u\, X_{H_c}$$

with $X_{H_i} = \begin{pmatrix} 0_n & I_n \\ -I_n & 0_n \end{pmatrix} \frac{\partial H_i}{\partial x}(x)$

- there exists structure preserving state feedback : $u = f\,(H_c)$ where H_c is the control Hamiltonian
- with closed-loop Hamiltonian $H_{cl} = H_0 + \Phi\,(H_c)$.

Structure preserving control for port Hamiltonian systems

For Port Hamiltonian systems

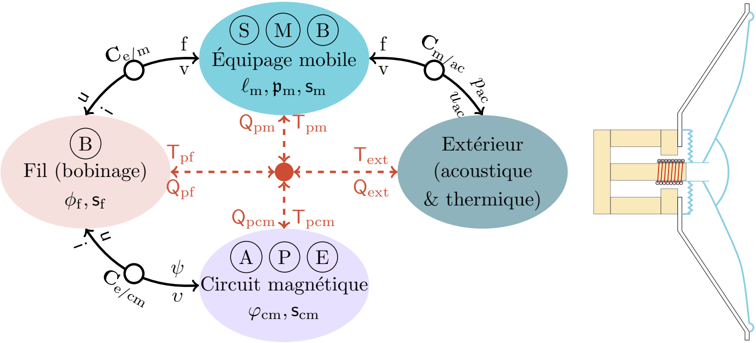
$$\dot{x} = [J(x) - R(x)]\frac{\partial H_0}{\partial x} + u\,g(x) \quad \textit{and} \quad y = g(x)\frac{\partial H_0}{\partial x}$$

the Interconnexion and Damping Assignment method assigns modified structure matrices J_{cl} , R_{cl} and Hamiltonian H_{cl} in closed loop for state-feedback $u(x)$ solution of a matching equation

$$-(J_a - R_a)\frac{\partial H_0}{\partial x}(x) + g(x)u(x) = [(J(x) + J_a(x)) - (R(x) + R_a(x))]\frac{\partial H_a}{\partial x}(x)$$

with design parameters
$J_a(x) = J_{\mathrm{cl}} - J(x),\ R_a(x) = R_{\mathrm{cl}} - R(x)$ and $H_a(x) = H_{\mathrm{cl}} - H_0(x)$.

Model of a loudspeaker with internal energy balance



[T. Lebrun, Ph.D. thesis IRCAM, 2019].

Ionic polymer metal composite *(IPMC)*

A polyelectrolyte gel (*electro-active polymers* (EAPs)) between metal electrodes

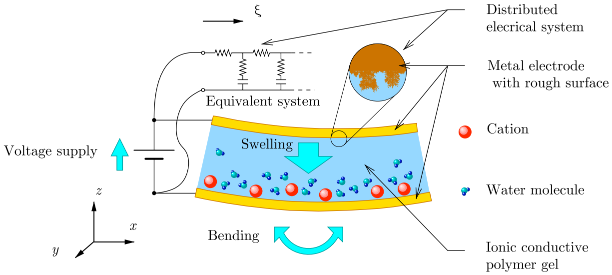


Fig. 2. Physical structure of IPMC.

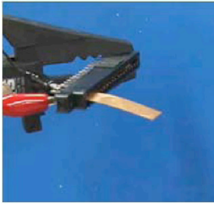


Fig. 1. IPMC (left:

G. Nishida, K. Takagi, B.M. Maschke and Z. Luo, Multi-Scale Distributed Parameter Modeling of Ionic Polymer-Metal Composite Soft Actuator, ***Control Engineering Practice****, Vol. 19, n°4, pp.321-334, 2011*

Structure-preserving control of dissipative Hamiltonian systems

- Assigning the structure matrices, Hamiltonian and irreversible entropy creation of *Irreversible port Hamiltonian systems* [Ramırez Estay et al., **Automatica**, 2016]
- Assigning the contact form, Hamiltonian and Legendre submanifold of *control contact Hamiltonian systems* [Ramırez Estay et al., **Systems and Control Letters**, 2013 ; **IEEE TAC**, 2017]

Irreversible Port Hamiltonian systems

An Irreversible Port Hamiltonian system (IPHS)

$$\dot{x} = J_{ir}\left(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}\right)\frac{\partial U}{\partial x}(x) + \underbrace{W\left(x, \frac{\partial U}{\partial x}\right) + g\left(x, \frac{\partial U}{\partial x}\right)u}_{\text{input map}}, \qquad (1)$$

$$J_{ir}\left(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}\right) = \underbrace{J_0(x)}_{\text{reversible}} + \underbrace{\gamma\left(x, \frac{\partial U}{\partial x}\right)\{S, U\}_J J}_{\text{irreversible coupling}} \qquad (2)$$

(i) $J_0(x)$ defines a Poisson bracket and J is a constant skew-symmetric matrix
(ii) $\gamma\left(x, \frac{\partial U}{\partial x}\right) > 0$ is a positive function (*second principle !*)
(iii) $U(x)$ is the Hamiltonian and $S(x)$ the entropy function which is a Casimir function of the Poisson structure matrix $J_0(x)$
(iii) $W\left(x, \frac{\partial U}{\partial x}\right)$, $g\left(x, \frac{\partial U}{\partial x}\right)$ are vector fields associated with the port.

In closed loop with $M(x) \geq 0$ and availability function (Bregman)
$A(x, x^*) = U(x) - U(x^*) - \frac{\partial U}{\partial x}(x^*)^\top (x - x^*)$

$$\dot{x} = \left(-\sigma_d M + \gamma_d \{S, A\}_{J_d} J_d\right)\frac{\partial A}{\partial x}$$

Structure preserving control for port Thermodynamic systems

We have seen the definition of Port Thermodynamic systems this morning and shall now answer the question of preserving feedback of Port Thermodynamic system .

- for which class of state-feedback $u(x)$ is the closed-loop system again a Port Thermodynamic system ?

In fact , the question may also be stated :

- when are 2 Port Thermodynamic systems state-feedback equivalent ?

Port Thermodynamical systems

Port Thermodynamical systems on the symplectized Thermodynamic Phase Space

The symplectization of the Thermodynamic Phase Space $x \in T^*\mathscr{X} \sim R^{2n}$ (P. Valentin and R. Balian)

Gibbs' relations written with respect to energy or entropy :

- energy form $d\,U = T\,d\,S - P\,d\,V + \mu\,d\,N$
- entropy form $d\,S = \frac{1}{T}\,d\,U + \frac{P}{T}\,d\,V - \frac{\mu}{T}\,d\,N$

which is rendered symmetric $p_U\,d\,U + p_S\,d\,S + p_V\,d\,V + p_N\,d\,N = 0$

Consider the symplectic manifold $T^*\mathscr{X}$ equiped with the canonical Liouville 1-form $\alpha = \sum_{i=0}^{n-1} p_i dq_i$ and symplectic 2-form $\omega = d\alpha$

The *thermodynamic phase space* $\mathbb{P}(T^*\mathscr{X})$ is obtained as the *projectivization* of $\mathscr{T}^*\mathscr{X}$ (the cotangent bundle $T^*\mathscr{X}$ *without* its zero-section) with contact form θ such that $\alpha = p_j\theta\,,\, j \in \{0, \ldots, n-1\}$

Homogeneous Control Hamiltonian systems on $\mathscr{T}^*\mathscr{X}$

Symplectic Space: $\mathscr{T}^*\mathscr{X}$ with canonical Liouville 1-form $\alpha = \sum_{i=0}^{n-1} p_i dq_i$ _
State space: Homogeneous Lagrange submanifold $L : \alpha_{|\mathfrak{L}} = 0$

A **Homogeneous control Hamiltonian system** is defined by:

- homogeneous in p H_0 internal and H_j interaction Hamiltonian : $K_{i|L} = 0$
- the differential equation: $\dot{x} = X_{H_0} + \sum_{j=1}^{m} u_j X_{H_j}$ with X_K a homogeneous symplectic Hamiltonian vector field: $L_{X_{H_j}} \alpha = 0$.

The physically relevant dynamics is the restriction to the Lagrangian invariant homogeneous submanifold of $\mathscr{T}^*\mathscr{X}$ or equivalently on the projection to a Legendre submanifold of $\mathbb{P}(T^*\mathscr{X})$.

Port Thermodynamic system on $\mathscr{T}^*\mathscr{X}$ (van der Schaft and Maschke, 2018)

Homogeneous Hamiltonian control system for which

- coordinate q_0^e corresponds to the total energy of the system
- coordinate q_1^e corresponds to the total entropy of the system
- the autonomous Hamiltonian satisfies

$$\left.\frac{\partial K^a}{\partial p_0^e}\right|_L = 0 \quad \text{and} \quad \left.\frac{\partial K^a}{\partial p_1^e}\right|_L \geq 0, \tag{3}$$

- augmented with the power-conjugated output $y_p = \left.\frac{\partial K^c}{\partial p_0^e}\right|_L$
- and the entropy-conjuguated output $y_p = \left.\frac{\partial K^c}{\partial p_1^e}\right|_L$

Structure preserving feedback

Structure preserving feedback

Characterization of Homogeneous Hamiltonian vector fields

Theorem

If the Hamiltonian function $K : T^*Q^e \to \mathbb{R}$ is homogeneous of degree 1 in p^e, then the Hamiltonian vector field $X = X_K$ satisfies

$$\mathbb{L}_X \alpha = 0 \tag{4}$$

where $\mathbb{L}_X$ denotes the Lie derivative with respect to the vector field X and α is the Liouville form. Conversely, if a vector field X satisfies (4) then $X = X_K$ for some locally defined Hamiltonian K that is homogeneous of degree 1 in p^e .

This is a stronger condition that the condition that the vector field X is (locally) Hamiltonian, consisting in leaving the symplectic form invariant $\mathbb{L}_X \omega = 0$

Feedback preserving the Liouville form

Theorem

Consider a homogeneous Hamiltonian control system and assume that the control Hamiltonian $K^c \in C^\infty(\mathscr{M})$ is zero on a submanifold of $\mathscr{T}^* Q^e$ with measure zero. Consider the feedback $u = \tilde{u}(q^e, p^e) \in C^\infty(T^* Q^e)$.
The closed-loop vector field

$$X = X_{K^a} + \tilde{u} X_{K^c} \tag{5}$$

is a Homogeneous Hamiltonian vector field if and only if the state feedback is constant, i.e., $\tilde{u}(q^e, p^e) = u_0 \in \mathbb{R}$.

Proof

Recall that a *Homogeneous* Hamiltonian vector field X satisfies

$$-i_X \omega = dK \quad \text{and} \quad i_X \alpha = K \tag{6}$$

Then using Cartan's formula one computes

$$\begin{aligned} \mathbb{L}_X \alpha &= \mathbb{L}_{(X_{K^a} + \tilde{u} X_{K^c})} \alpha \\ &= \underbrace{\mathbb{L}_{X_{K^a}} \alpha}_{=0} + \tilde{u} \underbrace{\left(i_{X_{K^c}} d\alpha \right)}_{=-dK^c} + d\left(\tilde{u} K_c \right) \\ &= K^c d\tilde{u} \end{aligned}$$

Hence the closed-loop vector field is again a homogeneous Hamiltonian vector field,implies that $\tilde{u}$ is a constant function.

Assigning a Pfaffian form in closed-loop

Theorem

The closed-loop vector field $X = X_{K^a} + \tilde{u} X_{K^c}$ *with feedback* $u = \tilde{u}(q^e, p^e)$ *is a homogeneous Hamiltonian vector field on* $\mathscr{T}^* Q^e$ *with respect to the Pfaffian form the added Pfaffian form* $\tilde{\alpha}$

$$\alpha_{cl} = \alpha + \tilde{\alpha}$$

if and only if
(i) the 2-form $\omega_{cl} = d\alpha_{cl}$ *is of rank* $2(n+1)$ *(hence it is a symplectic form)*
(ii) the following matching equation is satisfied

$$\left(L_{X_{K^a}} \tilde{\alpha}\right) + \tilde{u} \left(L_{X_{K^c}} \tilde{\alpha}\right) + \left(i_{X_{K^c}} \tilde{\alpha} + K^c\right) d\tilde{u} = 0 \qquad (7)$$

Proof

Let us check the closed-loop vector field satisfies $\mathbb{L}_X \alpha = 0$
Compute

$$L_X \alpha_{\text{cl}} = L_X (\alpha + \tilde{\alpha}) = K^c d\tilde{u} + L_X \tilde{\alpha}$$

and

$$\begin{aligned} L_X \tilde{\alpha} &= L_{(X_{K^a} + \tilde{u} X_{K^c})} \tilde{\alpha} \\ &= L_{X_{K^a}} \tilde{\alpha} + \tilde{u} \left(i_{X_{K^c}} d\tilde{\alpha} \right) + d \left(\tilde{u}\, i_{X_{K^c}} \tilde{\alpha} \right) \\ &= L_{X_{K^a}} \tilde{\alpha} + \tilde{u} \left(i_{X_{K^c}} d\tilde{\alpha} + d \left(i_{X_{K^c}} \tilde{\alpha} \right) \right) + \left(i_{X_{K^c}} \tilde{\alpha} \right) d\tilde{u} \\ &= L_{X_{K^a}} \tilde{\alpha} + \tilde{u} \left(L_{X_{K^c}} \tilde{\alpha} \right) + \left(i_{X_{K^c}} \tilde{\alpha} \right) d\tilde{u} \end{aligned} \tag{8}$$

leading to the matching equation (7).

Necessary matching equation

Corollary

The matching equation admits the necessary condition

$$0 = d\left(i_{X_{K^a}} d\tilde{\alpha}\right) + \tilde{u}\, d\left(i_{X_{K^c}} d\tilde{\alpha}\right) + \left(dK^c - i_{X_{K^c}} d\tilde{\alpha}\right) \wedge d\tilde{u} \qquad (9)$$

Proof.

he matching equation (7) is equivalent to

$$0 \quad = \quad L_{X_{K^a}} \tilde{\alpha} + \tilde{u}\left(i_{X_{K^c}} d\tilde{\alpha}\right) + d\left(\tilde{u}\left(i_{X_{K^c}} \tilde{\alpha}\right)\right) + K^c\, d\tilde{u}$$

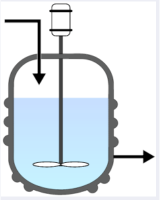
Computing its exterior derivative leads to

$$0 \quad = \quad d\left(i_X d\tilde{\alpha}\right) + dK^c \wedge d\tilde{u} \qquad (10)$$

where X is the closed-loop vector field □

Illustration on the model of a CSTR

Illustration on the model of a CSTR
[Maschke and van der Schaft, IFAC LHMNLC 2018]



Model of a CSTR

Continuous Stirred Tank reactor

- a mixture of two species A and B are highly diluted in an inert I
- a single chemical reaction $A \rightleftharpoons \beta B$ where β is a stoichiometric coefficientof the reaction
- a jacket in which a cooling fluid is at the temperature $T_w(t)$ being the control variable
- it is assumed that the inlet stream(with the *constant* volume flow rate $\mathfrak{Q}_I$) contains only the species A and the inert I.

Thermodynamic properties of the CSTR

The symplectified Thermodynamic Phase Space is

$$\mathbb{R}^8 \ni \tilde{x} = \left(q_S, q_U, q_{n_A}, q_{n_B}, \, p_S, p_U, p_{n_A}, p_{n_B}\right)^\top$$

Thermodynamic properties are defined by the Lagrangian submanifold generated by the function

$$G\left(U, n_A, n_B, p_S\right) = -p_S \, S\left(U, n_A, n_B\right) \tag{11}$$

where $S\left(U, n_A, n_B\right)$ is the *total entropy function* .

Definition of Hamiltonian functions for the CSTR (1)

Homogeneous Hamiltonian Control System $\dot{\tilde{x}} = X_{K^a} + T_w X_{H_{jK^c}}$ with

- drift Hamiltonian function

$$
\begin{aligned}
K^a &= h_0(U, n_A, n_B) + h_{flow}(U, n_A, n_B)\,\mathfrak{Q} \\
&\quad - \left(p_U + p_S \frac{\partial S}{\partial U}\right) \kappa \tilde{T}(U, n_A, n_B)
\end{aligned}
$$

- and control Hamiltonian function

$$
K^c = \left(p_U + p_S \frac{\partial S}{\partial U}\right) \kappa
$$

Definition of Hamiltonian functions for the CSTR (2)

- Internal Hamiltonian function (corresponding to the chemical reaction)

$$h_0 = \Pi\, r(T, n_A, n_B)\, V \begin{pmatrix} 0 \\ -1 \\ \beta \end{pmatrix} \tag{12}$$

$$= \left(-\left(p_{n_A} + p_S \frac{\partial S}{\partial n_A} \right) + \beta \left(p_{n_B} + p_S \frac{\partial S}{\partial n_B} \right) \right) r(T, n_A, n_B)\, V \tag{13}$$

- Hamiltonian function associated with constant inlet flow is

$$h_{flow} = \Pi \begin{pmatrix} \mathscr{C}_p^{in} \left(T^{in} - T_0\right) + \left(C_A^{in} h_{0_A} + C_I h_{0_I}\right) - \frac{1}{V}\tilde{H} \\ \frac{1}{V}\left(C_A^{in} V - n_A\right) \\ -n_B \end{pmatrix} \tag{14}$$

where

$$\Pi = \left(\left(p_U + p_S \frac{\partial S}{\partial U} \right), \left(p_{n_A} + p_S \frac{\partial S}{\partial n_A} \right), \left(p_{n_B} + p_S \frac{\partial S}{\partial n_B} \right) \right)$$

The matching eq. for the CSTR with temperature control

As an example let us choose as added Pfaffian form

$$\tilde{\alpha} = \varphi \, dq_S$$

where $\varphi \in C^\infty(\mathscr{T}^* Q^e)$.
The matching equation (7) is equivalent to

$$\begin{aligned} 0 \quad = \quad & [1+\kappa\tilde{u}] \left(i_{X_{K^c}} d\varphi \right) dq_S \\ & + \varphi \left[d \left(\frac{\partial h_0}{\partial p_S} + \frac{\partial h_{flow}}{\partial p_S} \right) + \tilde{u}\kappa d \left(\frac{\partial S}{\partial U} \right) \right] \\ & + \kappa \left(\frac{\partial S}{\partial U} \varphi + \left(p_U + p_S \frac{\partial S}{\partial U} \right) \right) d\tilde{u} \end{aligned}$$

The matching eq. for the CSTR with temperature control

Nullify the factor of dq_S, with functions φ satisfying $\left(i_{X_{K^c}} d\varphi\right) = 0$
Choosing

$$\varphi = -\left(\left(\frac{\partial S}{\partial U}\right)^{-1} p_U + p_S\right),$$

which ensures that the 2-form $\omega_{\mathrm{cl}} = d\alpha_{\mathrm{cl}}$ is of full rank.
The matching equation reduces to

$$d\left(\frac{\partial h_0}{\partial p_S} + \frac{\partial h_{flow}}{\partial p_S}\right) + \tilde{u}\kappa d\left(\frac{\partial S}{\partial U}\right) = 0 \tag{15}$$

By taking the exterior derivative one obtains the condition $d\tilde{u} \wedge d\left(\frac{\partial S}{\partial U}\right) = 0$, which implies that the control $\tilde{u}$ is a function of the reciprocal temperature $\frac{\partial S}{\partial U}$ which is a common assumption.

Case when the added 1-form is exact: $\alpha_{\mathrm{cl}} = \alpha + dF$

Consider the particular case, when the added 1-form $\tilde{\alpha}$ is exact;

$$\tilde{\alpha} = dF$$

with $F \in C^\infty(\mathscr{T}^* Q^e)$ being a a (smooth) real-valued function.
Then the closed-loop 1-form is changed to $\alpha_{\mathrm{cl}} = \alpha + dF$ but the closed-loop symplectic form is invariant $\omega_{\mathrm{cl}} = \omega$.
Then the necessary matching equation (9) reduces to

$$dK^c \wedge d\tilde{u} = 0$$

hence the state feedback is a function of the control Hamiltonian function

$$\tilde{u}(q^e, p^e) = \phi\left(K^c(q^e, p^e)\right)$$

with $\phi \in C^\infty(\mathbb{R})$. Very similar to input-output Hamiltonian systems !

Assigning a Pfaffian form in closed-loop with exact addded form

Proposition

The closed-loop vector field $X_{K^a} + \tilde{u} X_{K^c}$, with $\tilde{u} \in C^\infty(\mathscr{T}^* Q^e)$, is a homogeneous Hamiltonian vector field with 1-form α_{cl} and Hamiltonian K_{cl},

$$\alpha_{cl} = \alpha + dF \qquad \text{and} \qquad K_{cl} = K^a + \Phi(K^c) + \kappa,$$

where $F \in C^\infty(\mathscr{T}^* Q^e)$ and $\Phi \in C^\infty(\mathbb{R})$ and control $\tilde{u} = \Phi'(K^c)$,, if and only if F and Φsatisfy the matching equation

$$dF(X_{K^a}) + \Phi'(K^c)\left[K^c + dF(X_{K^c})\right] - \Phi(K^c) = \kappa \qquad (16)$$

Proof

Using again Cartan's formula and $d\tilde{\alpha} = 0$, the matching equation (7) becomes

$$0 = d\left[i_{X_{K^a}} dF + \phi\left(K^c\right)\left(i_{X_{K^c}} dF\right)\right] + K^c \phi'\left(K^c\right) dK^c$$

By integration, there exist $\Psi \in C^\infty(\mathbb{R})$ and $\kappa \in \mathbb{R}$ such that

$$i_{X_{K^a}} dF + \phi\left(K^c\right)\left(i_{X_{K^c}} dF\right) = \Psi\left(K^c\right) + \kappa$$

with $\Psi'(x) = -x\,\phi'(x)$

Then,one derives the closed-loop Hamiltonian function

$$\begin{aligned} K_{\mathrm{cl}} &= i_X \alpha_{\mathrm{cl}} = i_{(X_{K^a} + \tilde{u} X_{K^c})}\left(\alpha + dF\right) \\ &= K^a + \phi\left(K^c\right) K^c + \Psi\left(K^c\right) + \kappa \\ &= K^a + \Phi\left(K^c\right) + \kappa \end{aligned}$$

with Φ is a primitive function of ϕ.

Non-isothermal mass-spring-damper system

Non isothermal mass-spring-damper system

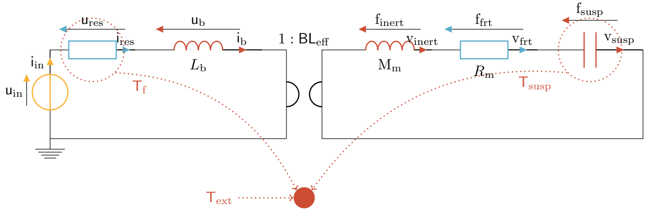


Figure: *Model of loudspeaker [T. Lebrun, Thèse doctorat , IRCAM, Paris, 2019]*

Model: state space

Consider Q^e with coordinates z (extension of the spring), π (momentum of the mass), E (total energy of the system) and S (the entropy of the system). The state space is the homogeneous Lagrangian submanifold $\mathscr{L} \subset T^*Q^e$

$$\begin{aligned} \mathscr{L} \;=\; & \{(z, \pi, S, E, p_z, p_\pi, p_S, p_E) \mid \\ & E = \tfrac{1}{2}kz^2 + \tfrac{\pi^2}{2m} + U(S), \\ & p_z = -p_E kz, p_\pi = -p_E \tfrac{\pi}{m}, p_S = -p_E U'(S)\} \end{aligned} \tag{17}$$

with spring constant k, mass m, and internal energy $U(S)$ and generating function

$$G = -p_E \left(\frac{1}{2}kz^2 + \frac{\pi^2}{2m} + U(S) \right)$$

Model: state space and Hamiltonian

The dynamics is generated by

- the autonomous Hamiltonian function is

$$K^a = p_z \frac{\pi}{m} + p_\pi \left(-kz - \nu \frac{\pi}{m} \right) + p_S \frac{\nu (\frac{\pi}{m})^2}{U'(S)}$$

- the control Hamiltonian function is

$$K^c = \left(p_\pi + p_E \frac{\pi}{m} \right)$$

which are homogeneous in the co-states !

Model: dynamics

The dynamics is with homogeneous Hamiltonian drift vector field and control vector field are

$$X_{K^a} = \begin{pmatrix} \frac{\pi}{m} \\ -kz - \nu\frac{\pi}{m} \\ \nu\frac{\pi}{m}\frac{1}{U'(S)} \\ 0 \\ k\,p_\pi \\ -\frac{p_z}{m} + p_\pi \nu \frac{1}{m} \\ p_S \nu \left(\frac{\pi}{m}\right)^2 \frac{U''(S)}{U'(S)^2} \\ 0 \end{pmatrix} \qquad X_{K^c} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{\pi}{m} \\ 0 \\ -\frac{p_E}{m} \\ 0 \\ 0 \end{pmatrix}$$

Matching equation and solution

Considering, as a simple example, $\Phi(x) = \frac{1}{2}x^2$, then the matching equation (16) becomes

$$\begin{aligned} \kappa &= dF(X_{K^a}) + K_c[K_c + dF(X_{K_c})] - \frac{1}{2}K_c^2 \\ &= dF(X_{K^a}) + K_c[\frac{1}{2}K_c + dF(X_{K_c})] \end{aligned}$$

It may be seen that there is a simple particular solution (for $\kappa = 0$)

$$F = -\frac{1}{2}\pi p_\pi - E p_E - \frac{1}{2} z p_z \tag{18}$$

Sructure preserving control

Equivalently, the nonlinear control $\tilde{u}(\pi, p_\pi, p_E) = \left(p_\pi + p_E \frac{\pi}{m}\right)$ and the added 1-form

$$\tilde{\alpha} \quad = \quad dF = -\frac{1}{2} p_z dz - \frac{1}{2} p_\pi d\pi - p_E dE$$

satisfy the matching equation (7).
Hence the closed-loop 1-form is

$$\begin{aligned} \tilde{\alpha} &= dF \\ &= -\frac{1}{2}\pi\, dp_\pi - E\, dp_E - \frac{1}{2} z\, dp_z - \frac{1}{2} p_z dz - \frac{1}{2} p_\pi d\pi - p_E dE \end{aligned}$$

and the closed-loop Hamiltonian is

$$\begin{aligned} K_{\mathrm{cl}} &= K^a + \Phi(K^c) \\ &= p_z \frac{\pi}{m} + p_\pi \left(-kz - \nu \frac{\pi}{m}\right) + p_S \frac{\nu \left(\frac{\pi}{m}\right)^2}{U'(S)} + \frac{1}{2}\left(p_\pi + p_E \frac{\pi}{m}\right)^2 \end{aligned}$$

Conclusion

Conclusion

Conclusion

We have considered Port Thermodynamic systems which are

- Homogeneous Hamiltonian systems
- defined on the symplectized Thermodynamic Phase Space,
- leaving a homogeneous Lagrangian submanifold invariant
- augmented with conjugated inputs and outputs: port variables

We have derived conditions for a state feedback to be structure preserving: matching equation between the added Pfaffian form and the control

Future work will be devoted to their control:

- stabilization
- synthesis of controller for particular classes: CSTR, etc..

Appendix

Appendix

Homogeneous *Lagrangian submanifolds* of T_0^*Q

Definition

A *homogeneous Lagrangian submanifold* $\mathscr{L} \subset T^*Q^e$ satisfies the two conditions

- it is a Lagrangian submanifold $\mathscr{L} \subset T^*Q^e$: it satisfies $\omega|_{\mathscr{L}} = 0$ and is maximal

- the homogeneity property:

$(q^e, p^e) \in \mathscr{L} \Rightarrow (q^e, \lambda p^e) \in \mathscr{L}$, for every $\lambda \in \mathbb{R}^*$

Alternatively, in [?] homogeneous Lagrangian submanifolds are geometrically characterized as maximal submanifolds satisfying $\alpha|_{\mathscr{L}} = 0$.

Relation between *Legendre submanifolds* of $\mathbb{P}(T^*Q)$ and *Lagrangian submanifolds* of T_0^*Q

Theorem

An integral submanifold N of θ is a Legendre submanifold of $\mathbb{P}(T^*Q)$if and only if $N_s := \pi^{-1}N$ is a Lagrangian submanifold of T_0^*Q with the projection $\pi : T_0^*Q \to \mathbb{P}(T^*Q)$.

To every Lagrangian submanifold L_s with *homogeneous generating function* of degree 1

$$G(q^0, \cdots, q^n, p_0, \cdots, p_n) = -p_0 S(q^1, \cdots, q^n)$$

there corresponds a Legendre submanifold L with generating function

$$G((q^0, \cdots, q^n, p_0, \cdots, p_n) = -p_0 F(q^I, \gamma_J)$$

where $\gamma_J = -\frac{p_J}{p_0}$

Relation between *Hamiltonian vector fields* of $\mathbb{P}(T^*Q)$ and of T_0^*Q

The contact vector fieldX_K on $\mathbb{P}(T^*Q)$ is the projection of the ordinary Hamiltonian vector field X_h on T_0^*Q

$$\pi * X_h = X_K$$

with h the Hamiltonian (homogeneous of degree 1) corresponding to the contact Hamiltonian K

$$h\left(q^0, q^1, \cdots, q^n, -1, \gamma 1, \cdot, \gamma_n\right) := K\left(q^0, q^1, \cdots, q^n, \gamma_1, \cdot, \gamma_n\right)$$

where $\gamma_J = -\frac{p_J}{p_0}$