Introduction

Background on Channel State Information (CSI)

- CSI data as seen from the baseband signal processing perspective is a random process in a high dimensional space (large number of receiving antennas, large sampling in the frequency domain, etc.).
- Physical considerations (e.g. ray-tracing channel models) hint at an unknown low-dimensional manifold structure of the CSI - roughly speaking, the CSI is a continuous function of a limited number of geometric and RF propagation parameters.

Objectives

- Learning the geometrical structure of CSI data while involving as little expert knowledge as possible.
- Data-driven identification of the parameterizations or features adapted to estimation, compression, prediction of CSI.

Dimensionality reduction of CSI

Assumptions on data manifold

- \( X \subset \mathbb{R}^D \) is a \( d \)-dimensional and continuous set in the Euclidean space \( \mathbb{R}^D \) with \( D \gg d \).
- We suppose that a generative model exists through some unknown continuous mapping \( g : \mathbb{R}^D \to X \) and seek for a mapping \( f : X \to \mathbb{R}^d \) so that \( f \approx g^{-1} \).

In practice, the observations \( x_i \) are noisy in the ambient space

Dimensionality Reduction

Principle: use distance between samples in order to learn \( f \approx g^{-1} \) up to an isometry.

General additional assumption on the sample indices \( S = \{1, \ldots, T\} \):

\[
\|w_i - w'_j\| = d(x_i, x_j) \quad \text{for all } (i, j) \in T \subset S \times S
\]

Non-parametric dimensionality reduction

- \( d(x_i, x_j) = \|x_i - x_j\| \quad T = S \times S \): multi-dimensional scaling (MDS).
- \( d(x_i, x_j) = \|x_i - x_j\| \quad \{ (i, j) \in S \times S : \|x_i - x_j\| \leq \eta \} \): Isomap, UMAP.

Main limitations

- Euclidean distance may not be adapted to the manifold \( X \).
- No explicit estimation of the mapping \( f \): difficult out-of-sample extrapolation.

Parametric dimensionality reduction: \( f(x_i) = \phi(x_i) \) with \( T = S \times S \)

- Distance in the ambient space parameterized by \( \theta \):

\[
d_{\theta}(x_i, x_j) = \|\phi(x_i) - \phi(x_j)\|
\]

Experimental Results

Experimental Dataset

Several hours of joint CSI/position measurements with a commercial user equipment (UE)

- In total, of \( 3.5 \times 10^8 \) CSI samples (sampling every 10ms).
- \( h_i \in \mathbb{C}^{64 \times 288} \) (64 receive antennas, 288 frequency subcarriers, \( 64 \times 288 = 18432 \)).

Expert pre-processing

- Some CSI characteristics are not of interest: e.g. clock and frequency offset between the UE and the base station.
- In order to circumvent these impairments and to reduce observation noise, we map the observations through a “projector” invariant wrt these characteristics.

\( x_i = p(h_i) \in \mathbb{R}^{1024} \) (i.e. \( D = 1024 \)).

Dimensionality Reduction Results (\( d = 2 \))

Semi-supervised extension

We generalize the triplets network criterion with known positions \( p_i \in \mathbb{R}^2 \):

\[
\theta = \arg \min_{\theta} \frac{1}{|T|} \sum_{(i, j, k) \in T} d_{\theta}(x_i, x_j) - d_{\theta}(x_i, x_k) + M \quad + \frac{\alpha}{|P|} \sum_{i \in P} \|\phi(x_i) - p_i\|
\]

Geographic position

Unsupervised \( (\alpha = 0, M = 1) \)

Semi-supervised \( (\alpha = 1, M = 1, |P| = 45) \)