Learning with Few Labeled Data

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Electrical and Systems Engineering &
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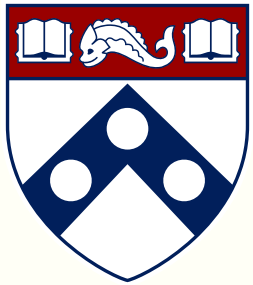


University of Pennsylvania

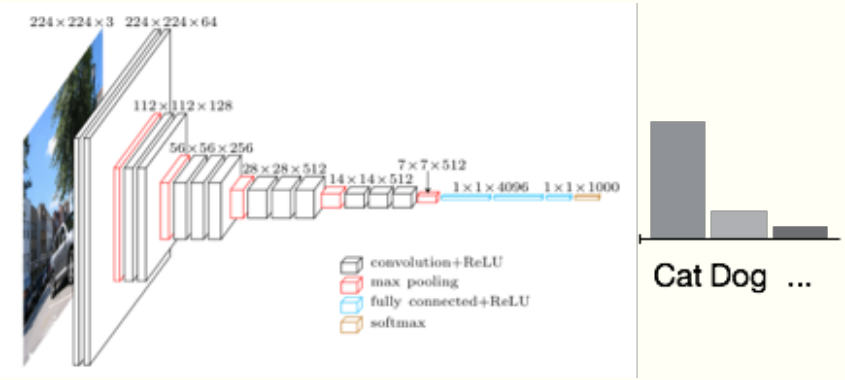
Menu

– **An introduction**

– Few-shot image classification

– Fundamental limits of representation learning

An introduction



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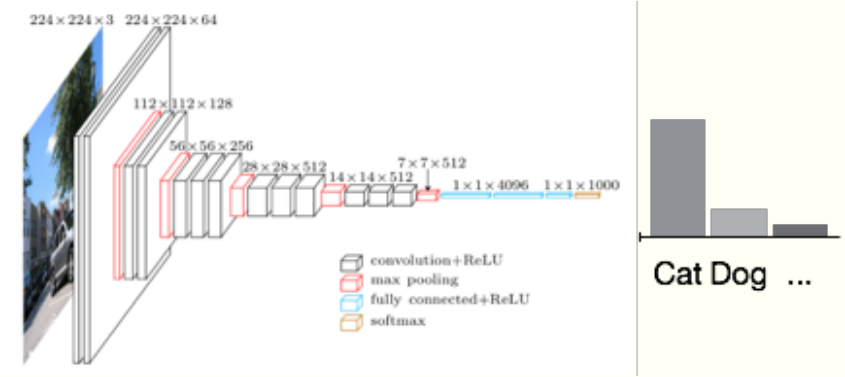

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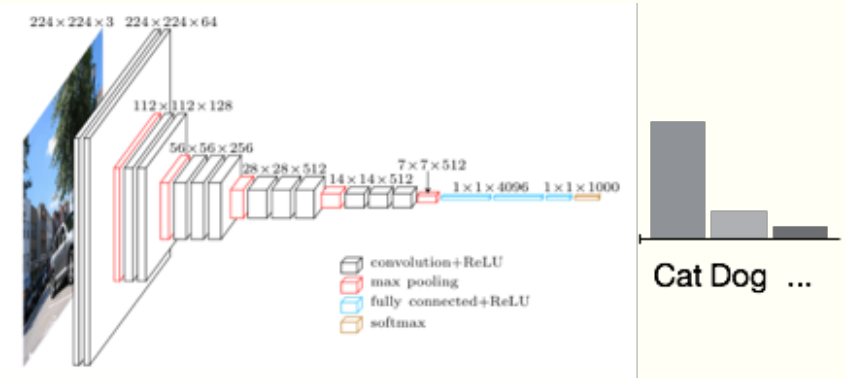

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In practice, Stochastic Gradient Descent (SGD) works extremely well

$$\theta_{t+1} = \theta_t - \frac{\eta}{\mathscr{b}} \sum_{k=1}^{\mathscr{b}} \nabla f_{\omega_k}(\theta_t)$$

$$\omega_k \in \{1, \ldots, N\}$$

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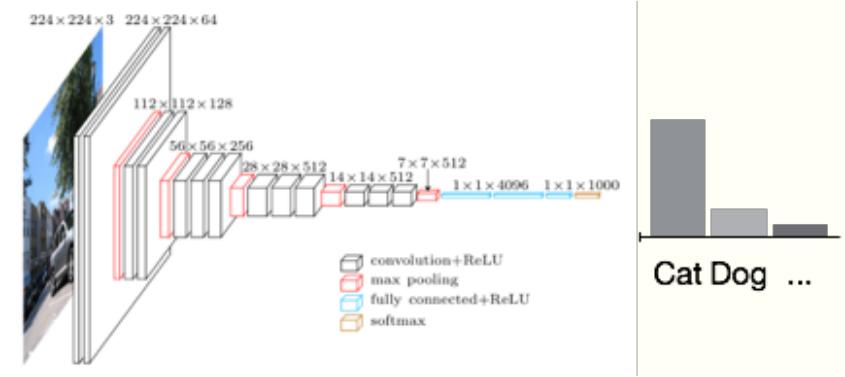

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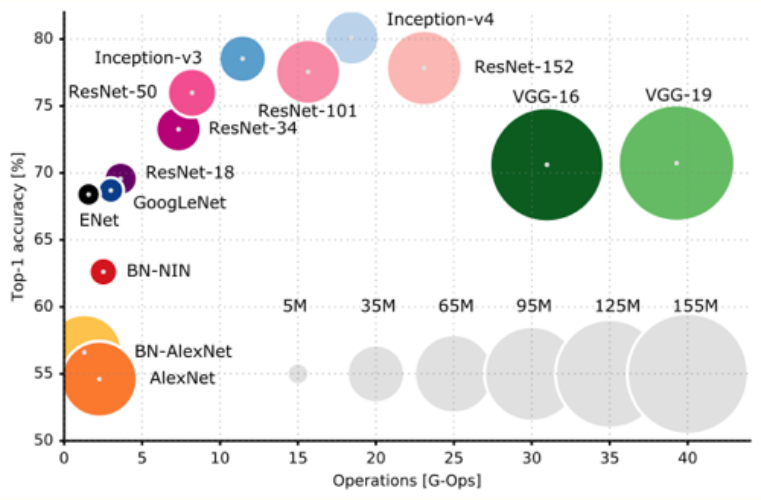
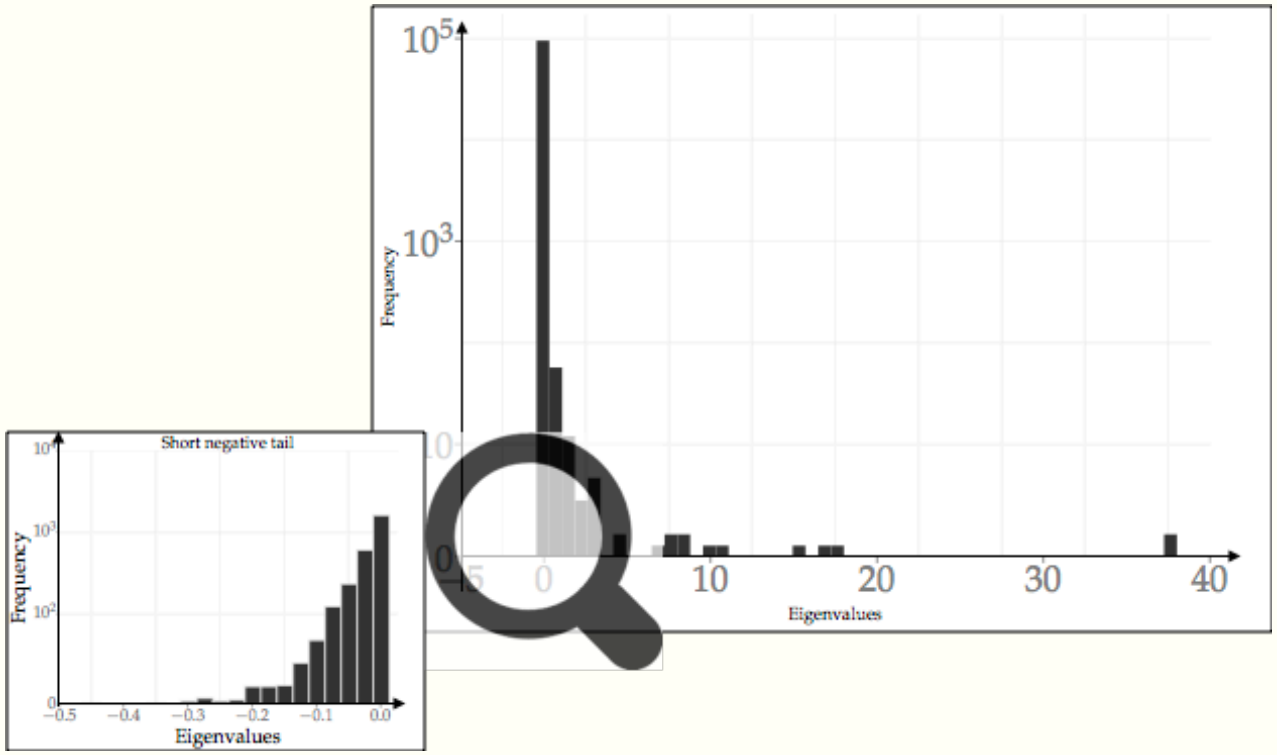


Why is SGD so special?

SGD finds “wide minima”



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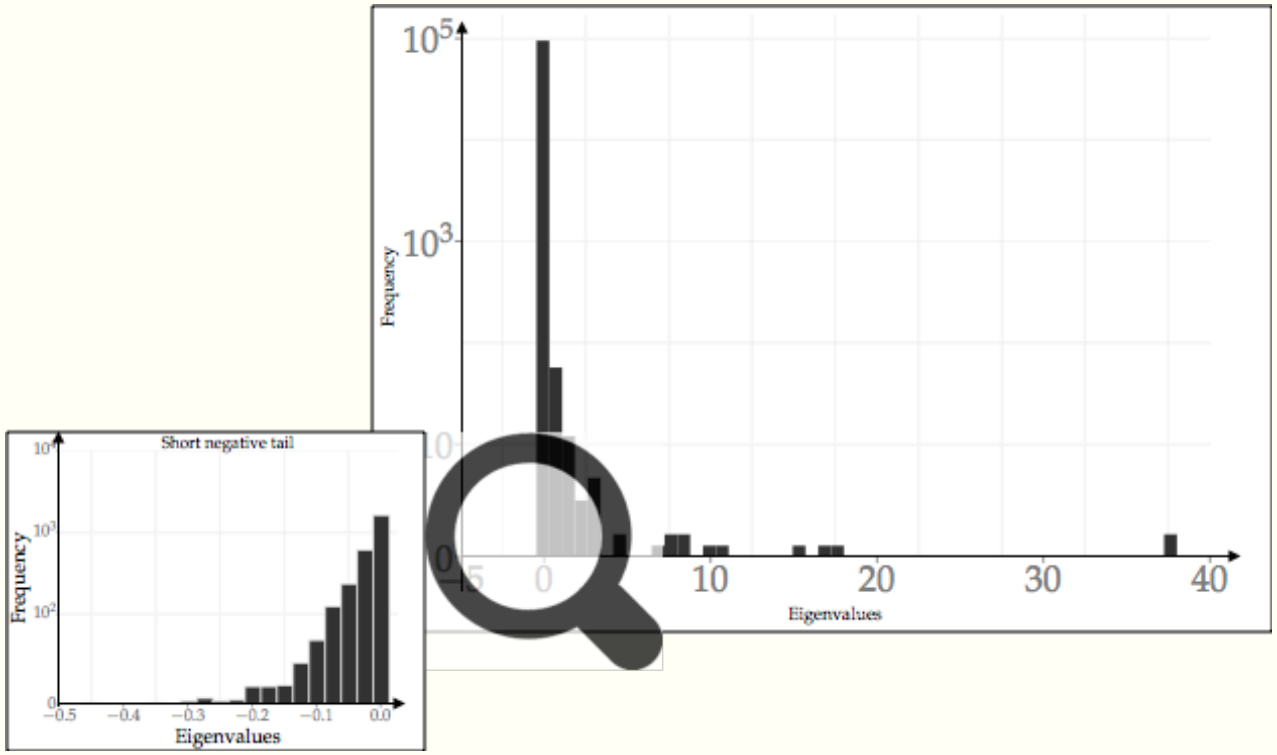
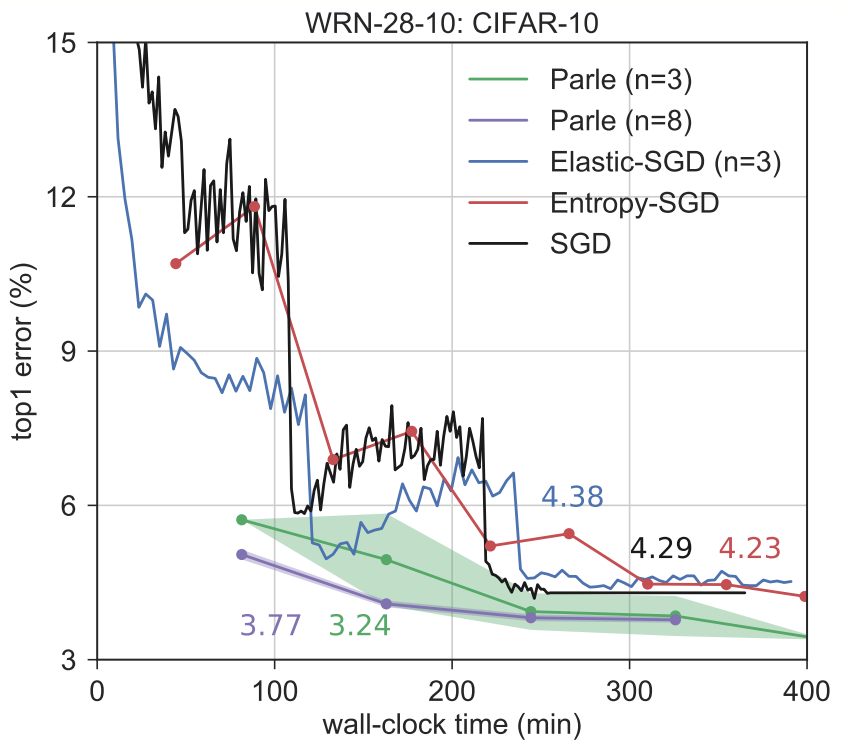


Local Entropy [Chaudhari et al., ICLR17] is a modified loss function for training deep networks

$$f_\gamma(\theta) = -\log(G_\gamma * e^{-f(\theta)})$$

Parle: parallelizing stochastic gradient descent

Couple MCMC and distributed updates to get state-of-the-art performance [Chaudhari et al., SysML18]



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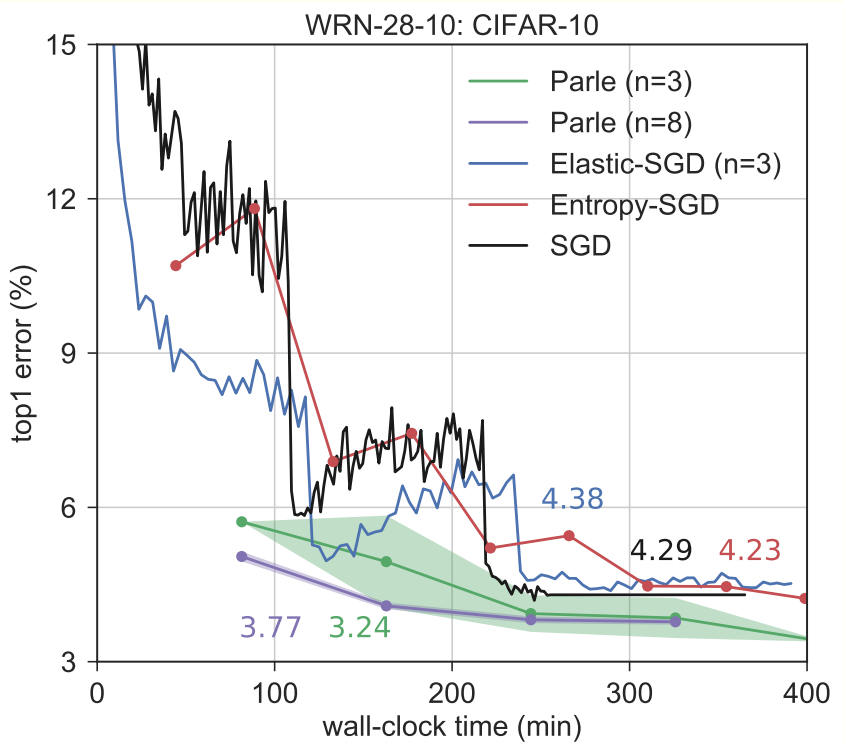

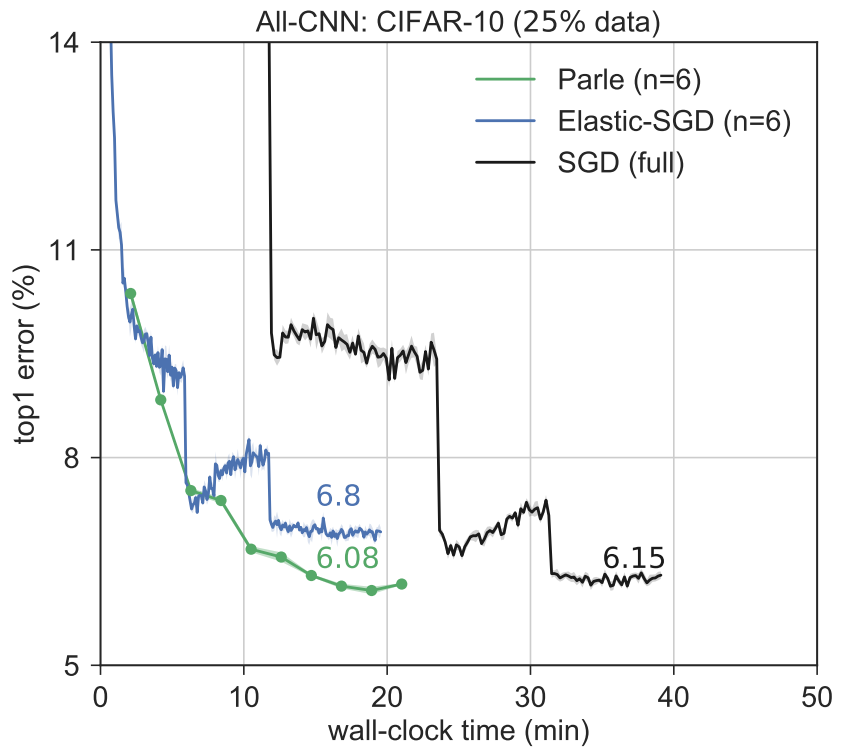

Diving deeper with statistical physics

A continuous-time analysis shows that SGD has a non-equilibrium steady-state distribution [Chaudhari & Soatto, ICLR 2018]

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$$\rho^{\text{ss}} = \operatorname*{argmin}_{\rho} \; \mathop{\mathbb{E}}_{\theta \sim \rho}[\Phi(\theta)] - \beta^{-1} H(\rho)$$

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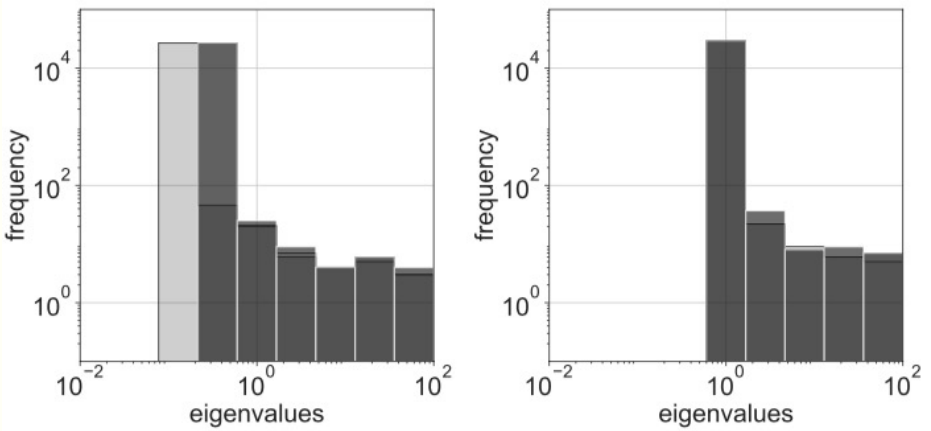
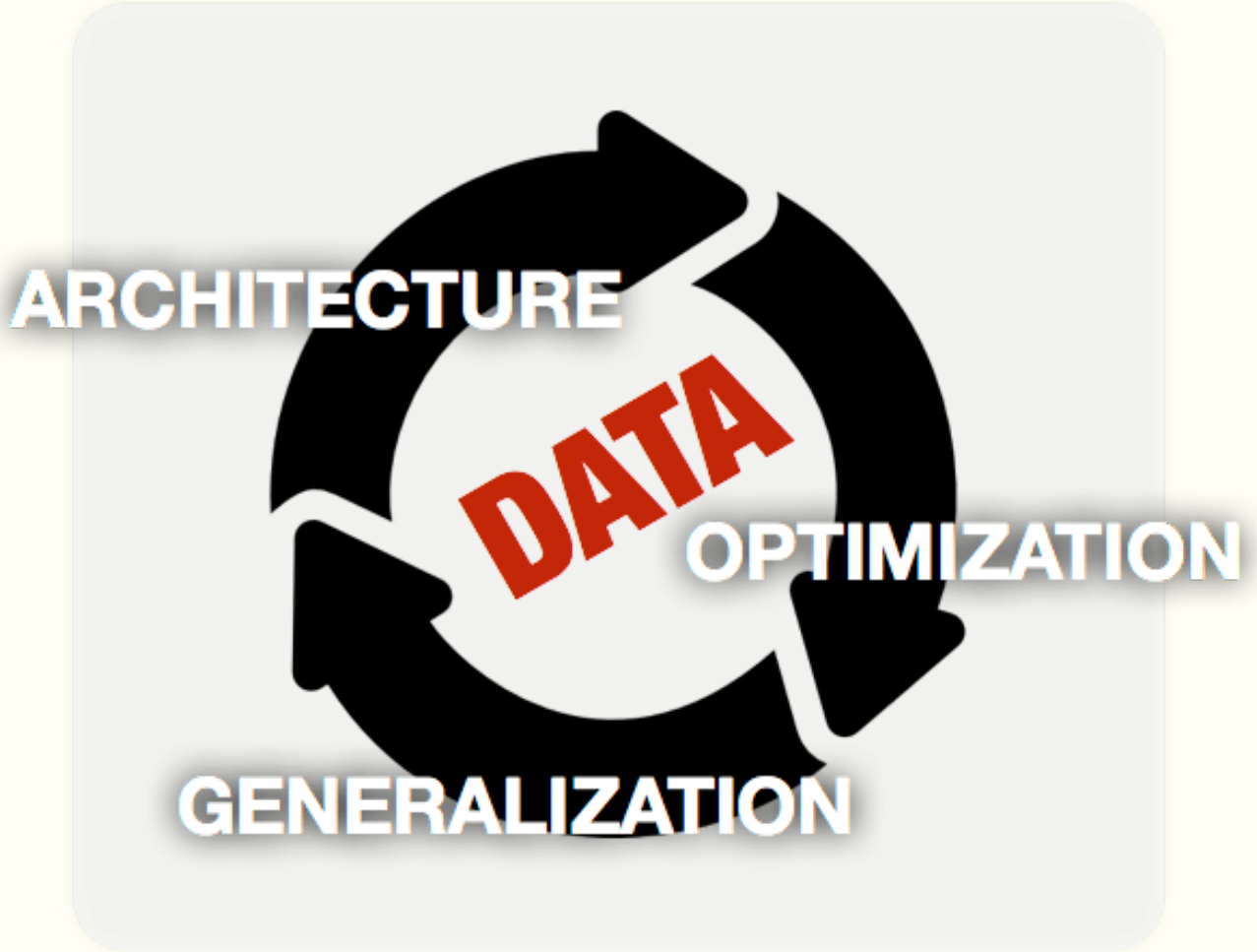
Noise in SGD is highly non-isotropic



(a) CIFAR-10
$\lambda(D) = 0.27 \pm 0.84$
$\text{rank}(D) = 0.34\%$

(b) CIFAR-100
$\lambda(D) = 0.98 \pm 2.16$
$\text{rank}(D) = 0.47\%$

Knots in our understanding



Menu

– An introduction

– **Few-shot image classification**

– Fundamental limits of representation learning

Three regimes of image classification

High-shot regime

100–1000 samples/class

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100–1000 samples/class

Low-shot regime

10 samples/class

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10 samples/class

Extreme low-shot regime (anomaly detection)

1 sample/class

Problem formulation

Training set consists of labeled samples from lots of "tasks", e.g., classifying cars, cats, dogs, planes . . .

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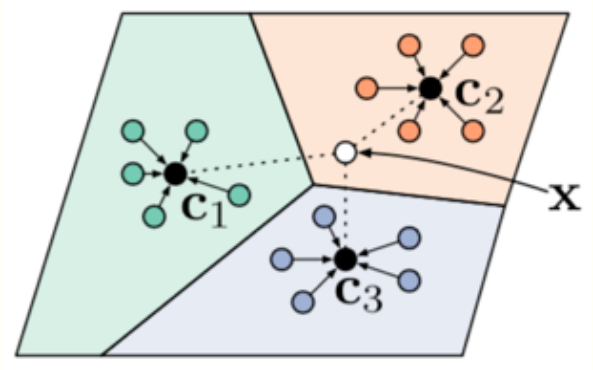
Few-shot setting considers the case when s is small.

A flavor of current few-shot algorithms

Prototypical Networks [Snell et al., 2017]

- Collect a meta-training set, this consists of a large number of related tasks
- Train one model on all these tasks to ensure that the clustering of features of this model correctly classifies the task
- If the test task comes from the same distribution as the meta-training tasks, we can use the clustering on the new task to classify new classes



The key idea

A classifier trained on a dataset D_s is a function F that classifies data x using

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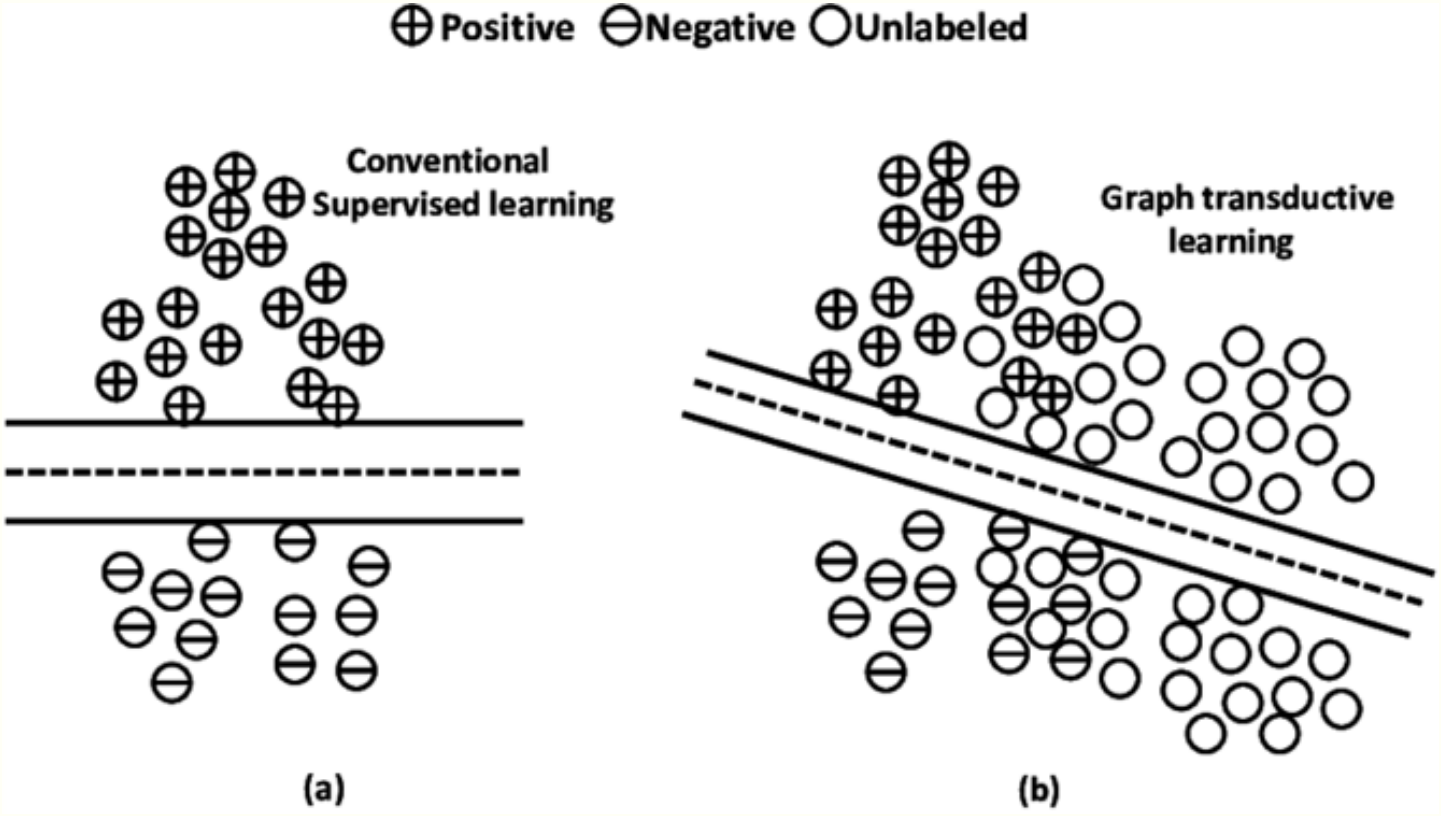
We cannot learn a good (sufficient) statistic using few samples. So we will search over functions at test-time more explicitly

$$\widehat{y} = \underset{y_{N_s+1}}{\operatorname{argmin}} \ \min_{\theta} \ \frac{1}{N_s+1} \sum_{i=1}^{N_s+1} -\log p_\theta(y_i \mid x_i) + \frac{1}{2\lambda}\|\theta - \theta^*(D_s)\|^2.$$

Transductive Learning



A very simple baseline

1. Train a large deep network on the meta-training dataset with the standard classification loss
2. Initialize a new "classifier head" on top of the logits to handle new classes
3. Fine-tune with the few labeled data from the new task
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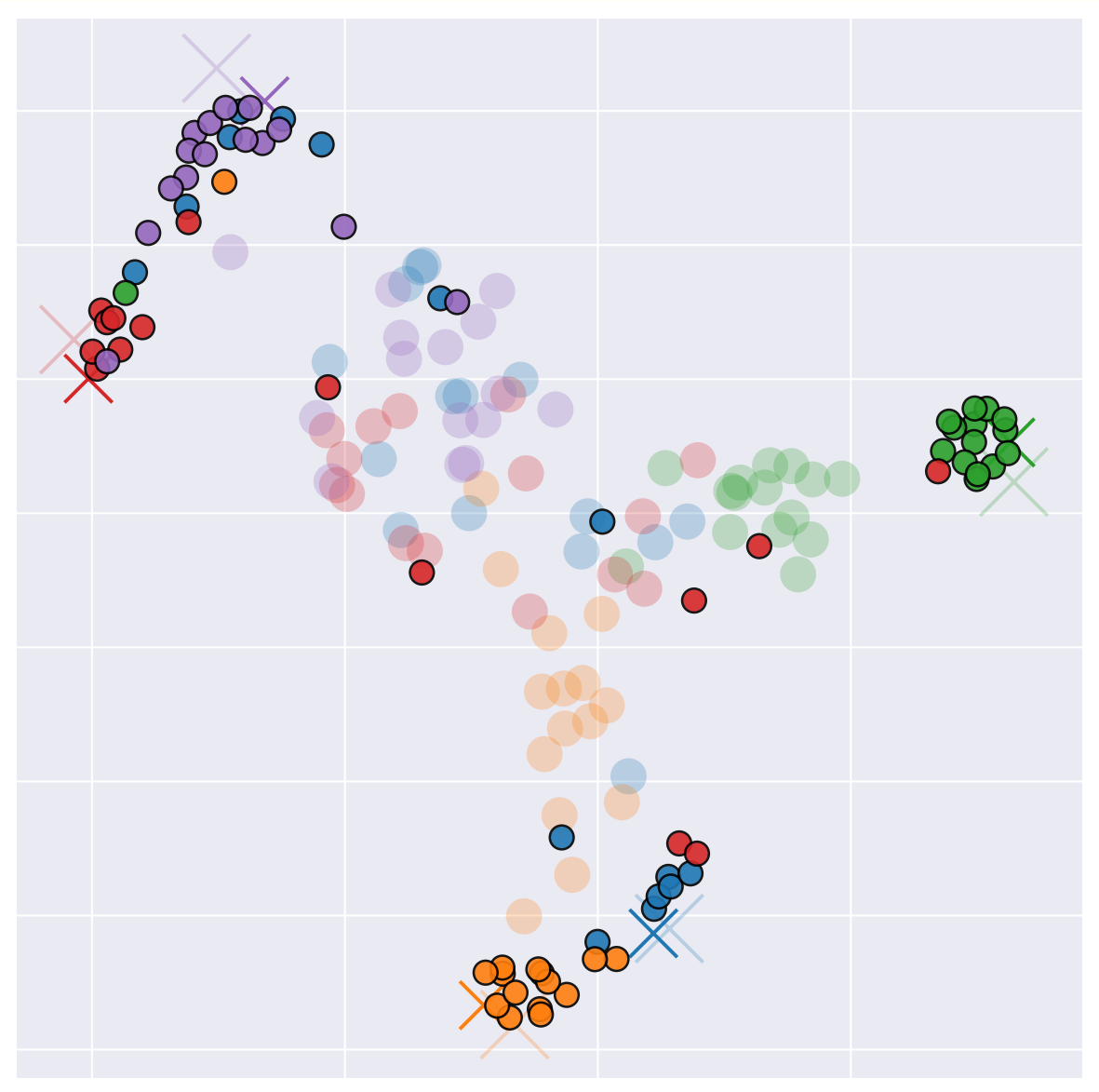
with a few practical tricks like cosine annealing of step-sizes, mixup regularization, 16-bit training, very heavy data augmentation, and label smoothing cross-entropy

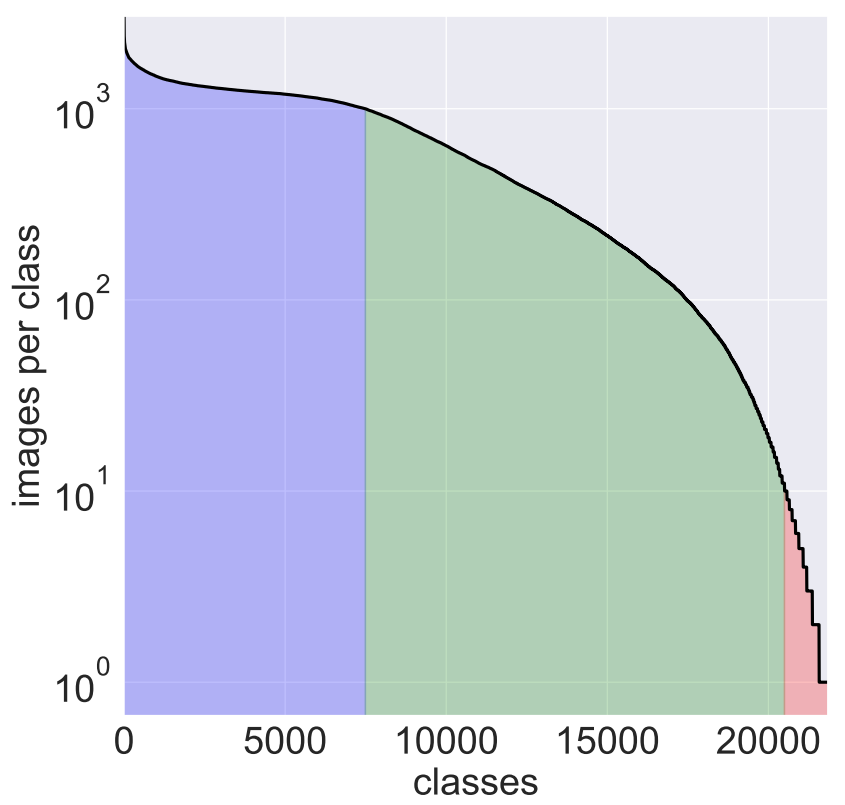
An example



Results on benchmark datasets

		Mini-ImageNet		Tiered-ImageNet		CIFAR-FS		FC-100	
Algorithm	Architecture	1-shot (%)	5-shot (%)	1-shot (%)	5-shot (%)	1-shot (%)	5-shot (%)	1-shot (%)	5-shot (%)
Matching networks (Vinyals et al., 2016)	conv $(64)_{\times 4}$	46.6	60						
LSTM meta-learner (Ravi & Larochelle, 2016)	conv $(64)_{\times 4}$	43.44 ± 0.77	60.60 ± 0.71						
Prototypical Networks (Snell et al., 2017)	conv $(64)_{\times 4}$	49.42 ± 0.78	68.20 ± 0.66						
MAML (Finn et al., 2017)	conv $(32)_{\times 4}$	48.70 ± 1.84	63.11 ± 0.92						
R2D2 (Bertinetto et al., 2018)	conv $(96^k)_{\times 4}$	51.8 ± 0.2	68.4 ± 0.2			65.4 ± 0.2	79.4 ± 0.2		
TADAM (Oreshkin et al., 2018)	ResNet-12	58.5 ± 0.3	76.7 ± 0.3					40.1 ± 0.4	56.1 ± 0.4
Transductive Propagation (Liu et al., 2018b)	conv $(64)_{\times 4}$	55.51 ± 0.86	69.86 ± 0.65	59.91 ± 0.94	73.30 ± 0.75				
Transductive Propagation (Liu et al., 2018b)	ResNet-12	59.46	75.64						
MetaOpt SVM (Lee et al., 2019)	ResNet-12 *	62.64 ± 0.61	$\mathbf{78.63 \pm 0.46}$	65.99 ± 0.72	81.56 ± 0.53	72.0 ± 0.7	84.2 ± 0.5	41.1 ± 0.6	55.5 ± 0.6
Support-based initialization (train)	WRN-28-10	56.17 ± 0.64	73.31 ± 0.53	$67.45 \pm 0.70^{\dagger}$	$82.88 \pm 0.53^{\dagger}$	70.26 ± 0.70	$83.82 \pm 0.49^{\dagger}$	36.82 ± 0.51	49.72 ± 0.55
Fine-tuning (train)	WRN-28-10	57.73 ± 0.62	$\mathbf{78.17 \pm 0.49}$	66.58 ± 0.70	$\mathbf{85.55 \pm 0.48}$	68.72 ± 0.67	$\mathbf{86.11 \pm 0.47}$	38.25 ± 0.52	$\mathbf{57.19 \pm 0.57}$
Transductive fine-tuning (train)	WRN-28-10	$\mathbf{65.73 \pm 0.68}$	$\mathbf{78.40 \pm 0.52}$	$\mathbf{73.34 \pm 0.71}$	$\mathbf{85.50 \pm 0.50}$	$\mathbf{76.58 \pm 0.68}$	$\mathbf{85.79 \pm 0.50}$	$\mathbf{43.16 \pm 0.59}$	$\mathbf{57.57 \pm 0.55}$
Activation to Parameter (Qiao et al., 2018) (train + val)	WRN-28-10	59.60 ± 0.41	73.74 ± 0.19						
LEO (Rusu et al., 2018) (train + val)	WRN-28-10	61.76 ± 0.08	77.59 ± 0.12	66.33 ± 0.05	81.44 ± 0.09				
MetaOpt SVM (Lee et al., 2019) (train + val)	ResNet-12 *	64.09 ± 0.62	$\mathbf{80.00 \pm 0.45}$	65.81 ± 0.74	81.75 ± 0.53	72.8 ± 0.7	85.0 ± 0.5	47.2 ± 0.6	62.5 ± 0.6
Support-based initialization (train + val)	WRN-28-10	58.47 ± 0.66	75.56 ± 0.52	$67.34 \pm 0.69^{\dagger}$	$83.32 \pm 0.51^{\dagger}$	$72.14 \pm 0.69^{\dagger}$	$85.21 \pm 0.49^{\dagger}$	45.08 ± 0.61	60.05 ± 0.60
Fine-tuning (train + val)	WRN-28-10	59.62 ± 0.66	$\mathbf{79.93 \pm 0.47}$	66.23 ± 0.68	$\mathbf{86.08 \pm 0.47}$	70.07 ± 0.67	$\mathbf{87.26 \pm 0.45}$	43.80 ± 0.58	64.40 ± 0.58
Transductive fine-tuning (train + val)	WRN-28-10	$\mathbf{68.11 \pm 0.69}$	$\mathbf{80.36 \pm 0.50}$	$\mathbf{72.87 \pm 0.71}$	$\mathbf{86.15 \pm 0.50}$	$\mathbf{78.36 \pm 0.70}$	$\mathbf{87.54 \pm 0.49}$	$\mathbf{50.44 \pm 0.68}$	$\mathbf{65.74 \pm 0.60}$

The ImageNet-21k dataset



1-shot, 5-way accuracies are as high as 89%, 1-shot 20-way accuracies are about 70%.

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– **Fundamental limits of representation learning**

Information Bottleneck Principle

A generalization of rate-distortion theory for learning representations of data [Tishby et al., 2000]

$$X \to Z \to Y$$

Z is a representation of the data X. We want

- Z to be sufficient to predict the target Y, and
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$$\min_{Z|X,\ Y|Z} \{I(X;Z) - I(Z;Y)\}.$$

Doing well on one task requires throwing away nuisance information [Achille & Soatto, 2017].

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The IB Lagrangian simply minimizes $I(X;Z)$, it does not let us measure what was thrown away.

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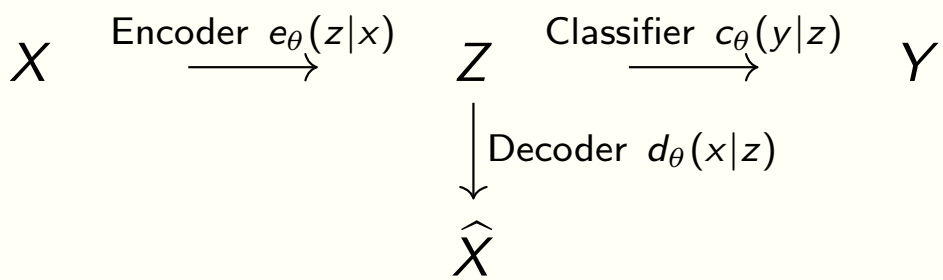
The architecture we will focus on is



An auto-encoder

Distortion D measures the quality of reconstruction

$$D = \mathop{\mathbb{E}}_{x \sim p(x)} \Big[- \int \mathrm{d}z \; e(z|x) \log d(x|z) \Big].$$

Rate R measures the average excess bits used to encode the representation

$$R = \mathop{\mathbb{E}}_{x \sim p(x)} \Big[\int \mathrm{d}z \; e(z|x) \log \frac{e(z|x)}{m(z)} \Big].$$

Rate-Distortion curve

The Lagrangian

$$F(\lambda) = \min_{e_\theta(z|x), m_\theta(z), d_\theta(x|z)} \{R + \lambda D\}.$$

is a relaxation of the fact that given a variational family and data there is an optimal value $R = \text{func}(D)$ (that best sandwiches the inequality $H - D \leq I(X; Z) \leq R$).

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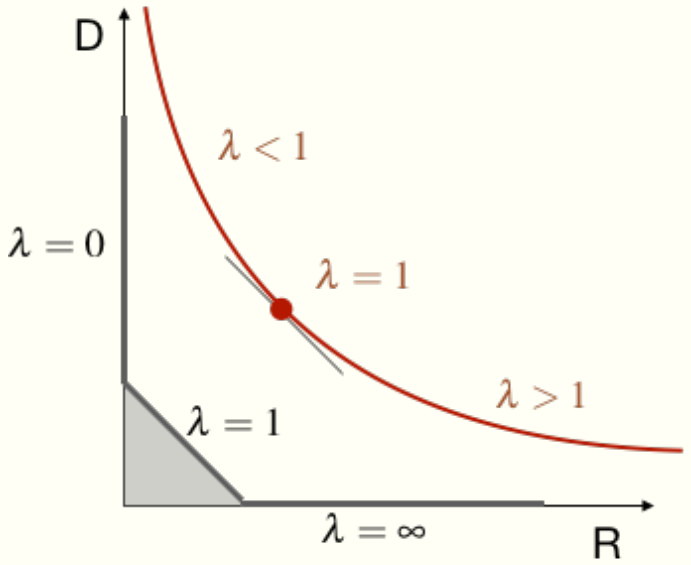

Rate-Distortion-Classification (RDC) surface

Let us extend the Lagrangian to

$$F(\lambda, \gamma) = \min_{e_\theta(z|x), m_\theta(z), d_\theta(x|z)} \{R + \lambda D + \gamma C\}$$

where the classification loss is

$$C = \mathop{\mathbb{E}}_{x \sim p(x), y \sim p(y|x)} \Big[- \int \mathrm{d}z \; e(z|x) \log c(y|z) \Big]$$

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Can also include other quantities like the entropy S of the model parameters

$$S = \mathop{\mathbb{E}}_{x\sim p(x),\; y\sim p(y|x)} \Big[\log \frac{p(\theta|\{x, y\})}{m(\theta)} \Big]$$

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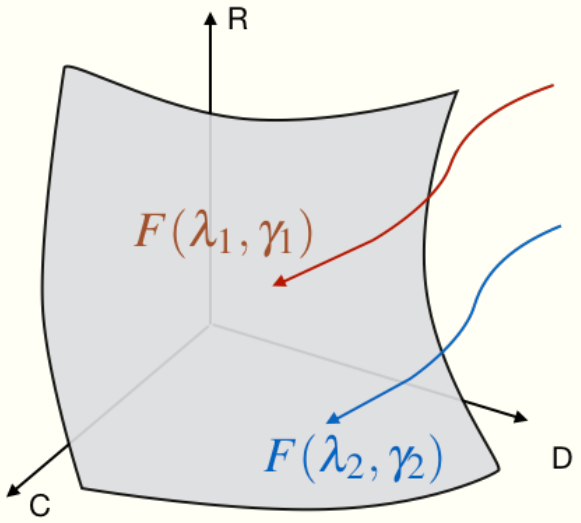

The existence of a convex surface $\text{func}(R, D, C, S) = 0$ tying together these functionals allows a formal connection to thermodynamics [Alemi & Fischer 2018]

$$\mathrm{d}R = -\lambda\ \mathrm{d}D - \gamma\ \mathrm{d}C - \sigma\ \mathrm{d}S.$$

Just like energy is conserved in physical processes, information is conserved in the model, either it is in the encoder-classifier pair or it is in the decoder.

Equilibrium surface of optimal free-energy

The RDC surface determines all possible representations that can be learnt from given data. Can solve the variational problem for $F(\lambda, \gamma)$ to get

$$Z_{\theta,x} = \int \mathrm{d}z \; m_\theta(z) \; d_\theta(x|z)^\lambda \; c_\theta(y_x|z)^\gamma$$

and

$$F(\lambda, \gamma) = \min_{\theta \in \Theta} \left\langle -\log Z_{\theta,x} \right\rangle_{x \sim p(x)} := J(\theta, \lambda, \gamma)$$

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The surface depends on data $p(x, y)$.

An iso-classification loss process

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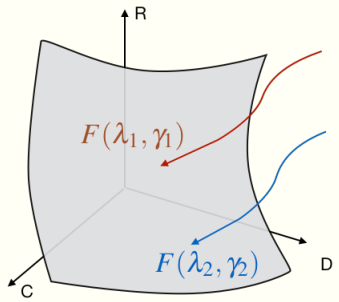
Can also impose other constraints, e.g.,

$$\frac{\mathrm{d}}{\mathrm{d}t}\{C + \gamma^{-1} R\} = 0$$

which is the objective for learning Bayesian neural networks.

Implementing processes on the RDC surface

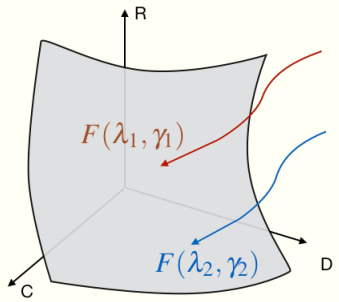


Could pick particular values of $(\dot{\lambda}, \dot{\gamma})$ to get

$$0 = \frac{\mathrm{d}}{\mathrm{d}t} \nabla_\theta J = \nabla_\theta^2 J \, \dot{\theta} + \dot{\lambda} \frac{\partial}{\partial \lambda} \nabla_\theta J + \dot{\gamma} \frac{\partial}{\partial \gamma} \nabla_\theta J$$

this requires inverting $\nabla_\theta^2 J$.

Implementing processes on the RDC surface



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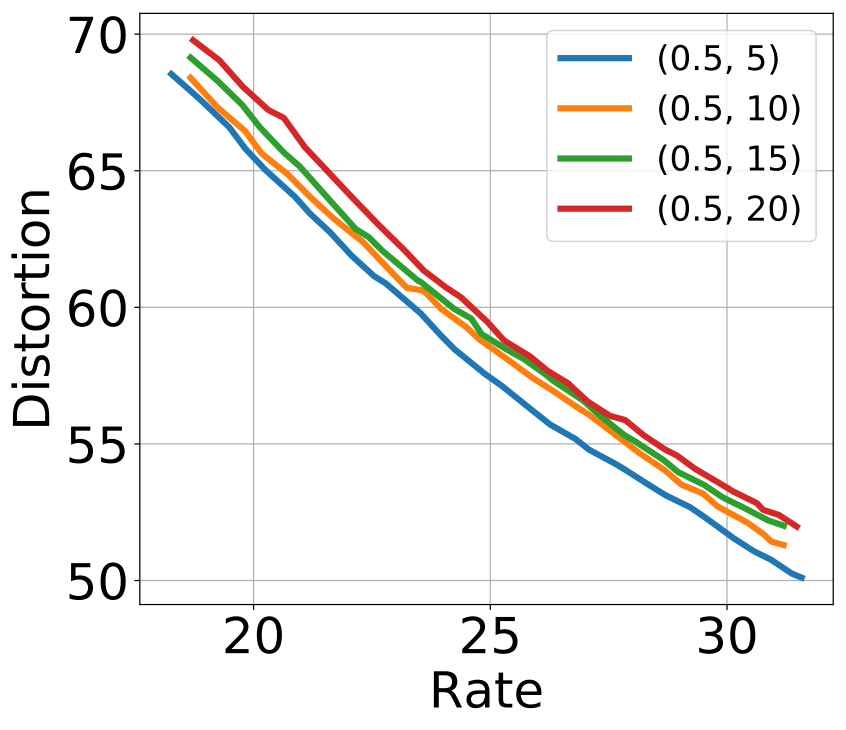
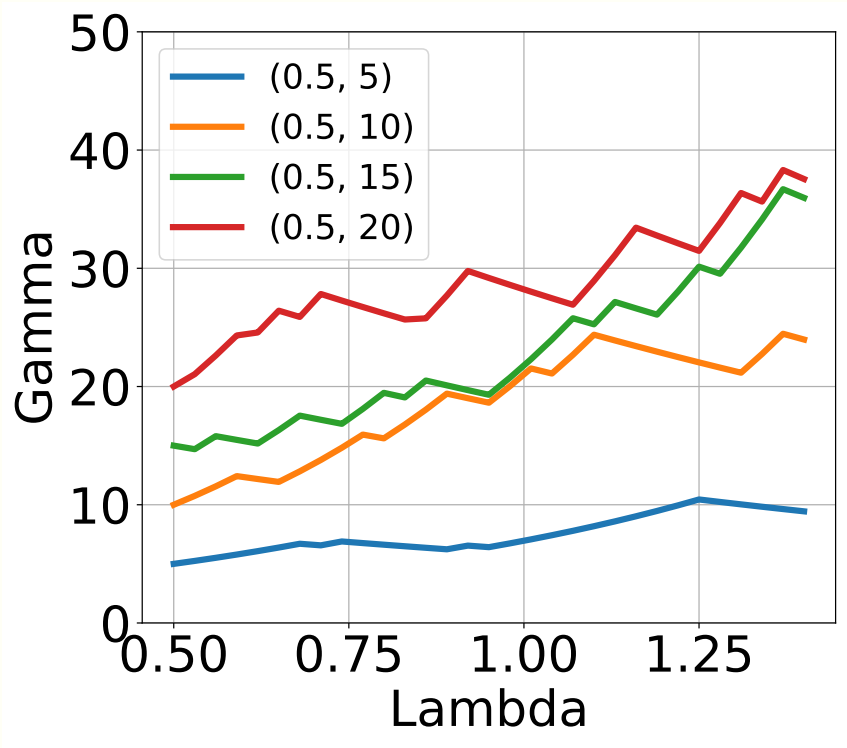
We exploit constraints like $0 = C_\lambda \dot{\lambda} + C_\gamma \dot{\gamma}$ to get

$$\dot{\lambda} = -\alpha \frac{\partial}{\partial \gamma} C = -\alpha \frac{\partial^2}{\partial \gamma^2} F$$

$$\dot{\gamma} = \alpha \frac{\partial}{\partial \lambda} C = \alpha \frac{\partial^2}{\partial \lambda \partial \gamma} F$$

Iso-C process for different initial (λ, γ): CIFAR-10





Transferring to new tasks

The RDC surface depends on data $p(x, y)$. We now move the data distribution from the source task to the target task, e.g., interpolate it as

$$p(x, y, t) = (1 - t)\ p^s(x, y) + t\ p^t(x, y).$$

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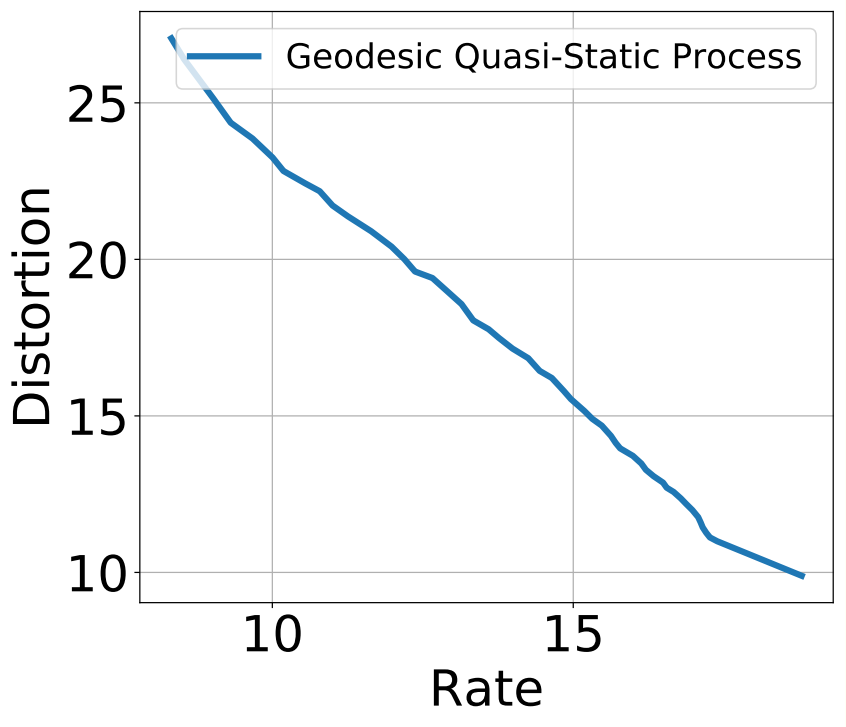
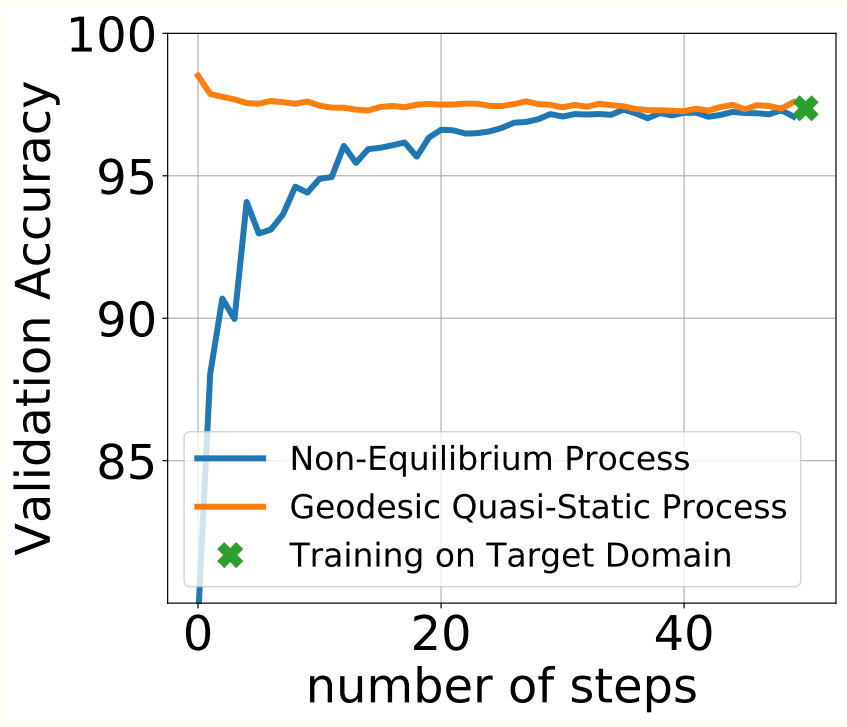
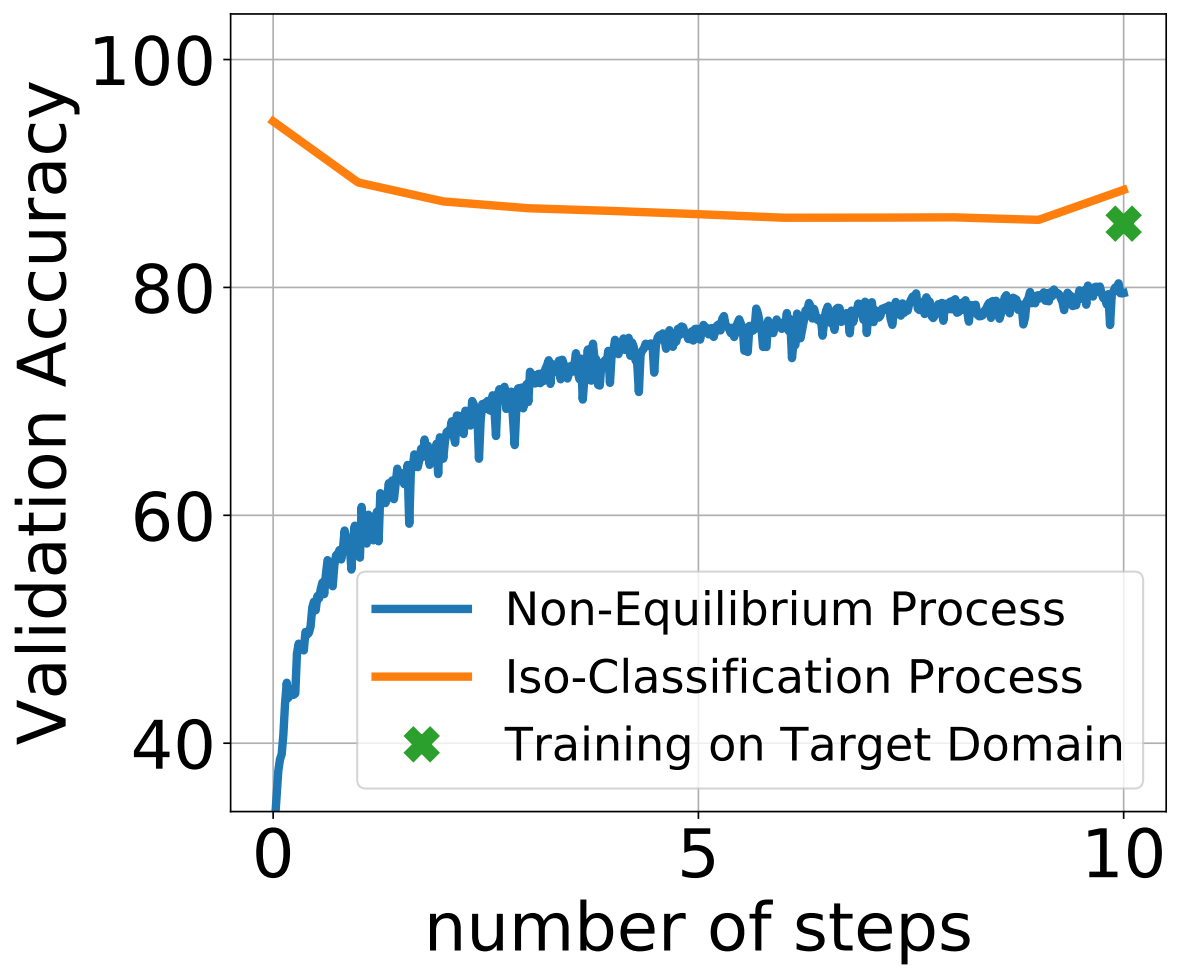
This is a completely controlled mechanism to transfer representations.

Iso-C process: MNIST 0–4 to 5–9





Iso-C process: CIFAR-10 Vehicles to Animals



Summary

Simple methods such as transductive fine-tuning work extremely well for few-shot learning. This is really because of powerful function approximators such as neural networks.

The RDC surface is a fundamental quantity and enables principled methods for transfer learning. Also unlocks new paths to understanding regularization and properties of neural architecture for classical supervised learning.

We did well in the era of big data without understanding much about data; this is unlikely to work in the age of little data.

Email questions to pratikac@seas.upenn.edu

Read more at

1. Chaudhari, P. and Soatto, S.. Stochastic gradient descent performs variational inference, converges to limit cycles for deep networks. arXiv:1710.11029. ICLR 2018.
2. Dhillon, G., Chaudhari, P., Ravichandran, A., and Soatto, S. A baseline for few-shot image classification. arXiv:1909.02729. ICLR 2020.
3. Gao, Y., and Chaudhari, P. A free-energy principle for representation learning.

   arXiv:2002.12406. ICML 2020