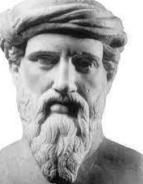
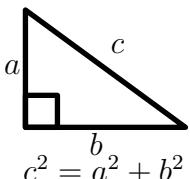
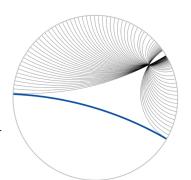
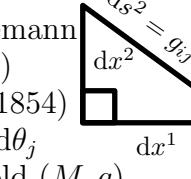
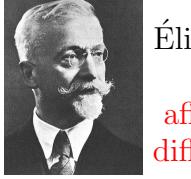


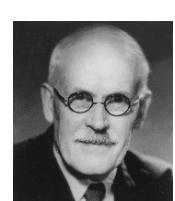
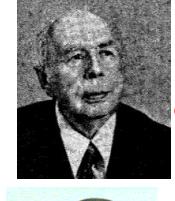
Genesis of the Dual Structure of Information Geometry

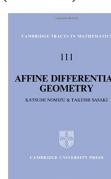
 Pythagoras of Samos (c. 570-495 BC)  Pythagoras' theorem  Euclid (ca 365-300 BC) Elements, math. proof Playfair axiom, Euclidean geometry

 Nikolai Ivanovich Lobachevsky (1792-1856)  Hyperbolic geometry (∞ -many lines passing through a point and // to another line)

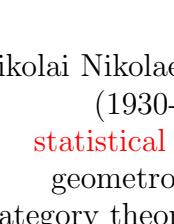
 Johann C. F. Gauss (1777-1855) differential geometry of surfaces Theorema Egregium  Georg F. B. Riemann (1826-1866) metric tensor (1854) $g = g_{ij} d\theta_i \otimes d\theta_j$  Riemannian manifold (M, g)

 Christian Felix Klein (1849-1925) Projective geometry & symmetry group Erlangen program  Élie Joseph Cartan (1869-1951) affine connections differential forms ω

 Sir Ronald Aylmer Fisher (1890-1962) Mathematical statistics Fisher information, MLE $I(\theta) = E_{p_\theta} [(\nabla_\theta \log p_\theta)(\nabla_\theta \log p_\theta)^\top]$  Sir Harold Jeffreys (1891-1989) Jeffreys prior $\propto \sqrt{|g|}$ J -divergence  Alexander P. Norden (1904-1993) conjugate connections wrt g Affinely connected spaces

 Harold Hotelling (1895-1973) Econometrician Fisher metric (1930)  Calyampudi Radhakrishna Rao (1920-) Fisher-Rao distance Cramér-Rao lower bound (1945)  Prasanta C. Mahalanobis (1893-1972) Distances in statistics Mahalanobis distance statistical field (1936)  Ernest Borisovich Vinberg (1937-2020) characteristic functions on homogeneous cones 

 Claude Elwood Shannon (1916-2001) Information theory Entropy: $h(p) = -\int p \log p d\mu$   Solomon Kullback (1907-1994) Richard A. Leibler (1914-2003) KL divergence $D_{KL}[p : q] = \int p \log \frac{p}{q} d\mu$  Imre Csiszár (1938-) information projections f -divergences $I_f[p : q] = \int p f\left(\frac{q}{p}\right) d\mu$  Jean-Louis Koszul (1921-2018) Hirohiko Shima homogeneous bounded domains 

 Nikolai Nikolaevich Chentsov (1930-1992) statistical invariance geometrostatistics Category theory, connections  Ole E. Barndorff-Nielsen (1935-) Exponential families observed information geometry  Shun-ichi Amari (1936-) Information geometry dualistic structure (M, g, ∇, ∇^*) : $Zg(X, Y) = g(\nabla_Z X, Y) + g(X, \nabla_Z^* Y)$ dual $\pm\alpha$ -connections $(M, g_F, \nabla^{-\alpha}, \nabla^\alpha)$  Bradley Efron (1938-) statistical curvature E -connection $D(P : Q) + D(Q : R) = D(P : R)$ Generalized Pythagoras' theorem in dually flat space (M, g, ∇, ∇^*)  Steffen Lauritzen (1947-) statistical manifold (M, g, C)  Information geometry journal (2018-)