

Geometric GSI'19 Science of Information

Toulouse, 27th - 29th August 2019

// Program



// Welcome message from GSI'19 chairmen



// **Frank Nielsen, co-chair**
Ecole Polytechnique, Palaiseau, France
Sony Computer Science Laboratories,
Tokyo, Japan



// **Frédéric Barbaresco, co-chair**
President of SEE ISIC Club (Ingénierie des
Systèmes d'Information et de Communi-
cations)
Representative of KTD PCC (Key Techno-
logy Domain / Processing, Computing &
Cognition) Board,
THALES LAND & AIR SYSTEMS, France

On behalf of both the organizing and the scientific committees, it is our great pleasure to welcome all delegates, representatives and participants from around the world to the fourth International SEE conference on "Geometric Science of Information" (GSI'19), hosted at ENAC in Toulouse, 27th to 29th August 2019.

GSI'19 benefits from scientific sponsor and financial sponsors.

The 3-day conference is also organized in the frame of the relations set up between SEE and scientific institutions or academic laboratories: ENAC, Institut Mathématique de Bordeaux, Ecole Polytechnique, Ecole des Mines ParisTech, INRIA, CentraleSupélec, Institut Mathématique de Bordeaux, Sony Computer Science Laboratories.

We would like to express all our thanks to the local organizers (ENAC, IMT and CIMI Labex) for hosting this event at the interface between Geometry, Probability and Information Geometry.

The GSI conference cycle has been initiated by the Brillouin Seminar Team as soon as 2009. The GSI'19 event has been motivated in the continuity of first initiatives launched in 2013 at Mines ParisTech, consolidated in 2015 at Ecole Polytechnique and opened to new communities in 2017 at Mines ParisTech. We mention that in 2011, we organized an indo-french workshop on "Matrix Information Geometry" that yielded an edited book in 2013, and in 2017, collaborate to CIRM seminar in Luminy TSGI'17 "Topological & Geometrical Structures of Information". Last GSI'17 Proceedings have been edited by SPRINGER in Lecture Notes and the most important contributions to the conference GSI'2017 have been collected in SPRINGER Book "Geometric Structures of Information".

The technical program of GSI'19 covers all the main topics and highlights in the domain of "Geometric Science of Information" including Information Geometry Manifolds of structured data/information and their advanced applications. This proceeding consists solely of original research papers that have been carefully peer-reviewed by two or three experts before, and revised before acceptance.

The GSI'19 program includes a renown honorary speaker, one guest honorary speaker and three keynote distinguished speakers, and an History session with a talk on "Pierre de FERMAT (ca. 1605-1665): lawyer, philologist and illustrious mathematician ... but enigmatic".

// GSI'19 conference "Geometric Science of Information"

As for GSI'13, GSI'15 and GSI'17, the objective of this SEE GSI'19 conference, hosted in Toulouse at ENAC, is to bring together pure/applied mathematicians and engineers, with common interest for Geometric tools and their applications for Information analysis. It emphasizes an active participation of young researchers to discuss emerging areas of collaborative research on "Geometric Science of Information and their Applications". Current and ongoing uses of Information Geometry Manifolds in applied mathematics are the following: Advanced Signal/Image/Video Processing, Complex Data Modeling and Analysis, Information Ranking and Retrieval, Coding, Cognitive Systems, Optimal Control, Statistics on Manifolds, Topology/Machine/Deep Learning, Artificial Intelligence, Speech/sound recognition, natural language treatment, Big Data Analytics, Learning for Robotics, etc., which are substantially relevant for industry.

The Conference will be therefore held in areas of topics of mutual interest with the aim to:

- Provide an overview on the most recent state-of-the-art
- Exchange mathematical information/knowledge/expertise in the area
- Identify research areas/applications for future collaboration

This conference will be an interdisciplinary event and will unify skills from Geometry, Probability and Information Theory.

Proceedings are published in Springer's Lecture Note in Computer Science (LNCS) series.

// CIMI Semester « Statistics with Geometry and Topology »

This thematic trimester aims to highlight recent advances and scientific synergies between statistics and geometry. The dynamics of this scientific combination between statistics and geometry is driven by many applications in the field of signal processing (radar, images, ...) and massive data (internet databases, monitoring, ...). This thematic trimester will undoubtedly be a scientific springboard for the development of a Statistical Geometry-Computational Geometry research axis, whose efflorescence is highly probable in the next decade. The trimester will open at the end of August 2019, with the GSI 2019 conference organized at ENAC. The thematic trimester will then extend, from September to November, around the three following scientific axis:

- Information Geometry (30th of August-6th of September 2019—14th-19th of October 2019),
- Topology for learning and data analysis (30th of September-4th of October 2019),
- Computational algebraic geometry, optimization and statistical applications (6th-8th November 2019).

Precise information of the conference and workshops:

- Aug 27 – Aug 29 Geometric Science of Information (GSI 19)
- Aug 30 – Sep 6 Geometric Statistics
- Sep 29 – Sep 4 Topology for Learning and Data Analysis
- Oct 14 – Oct 19 Information Geometry
- Nov 6 – Nov 8 Computational Aspects of Geometry

Local organizers:

F. Costantino, F. Gamboa, D. Henrion, T. Klein, A. Le Brigant, F. Nicol, E. Pauwels

Scientific committee:

M. Arnaudon, F. Barbaresco, J. Bigot, A. Garivier, J-B. Lasserre, X. Pennec, S. Puechmorel

// Tribute to Jean-Louis Koszul
(1921 – 2018)

Jean-Louis Koszul passed away January 12th 2018. This tribute is a scientific exegesis and admiration of Jean-Louis Koszul's works on homogeneous bounded domains that have appeared over time as elementary structures of Information Geometry. Koszul has introduced fundamental tools to characterize the geometry of sharp convex cones, as Koszul-Vinberg characteristic Function, Koszul Forms, and affine representation of Lie Algebra and Lie Group. The 2nd Koszul form is an extension of classical Fisher metric. Koszul theory of hessian structures and Koszul forms could be considered as main foundation and pillars of Information Geometry.

The community of "Geometric Science of Information" (GSI) has lost a mathematician of great value, who informed his views by the depth of his knowledge of the elementary structures of hessian geometry and bounded homogeneous domains. His modesty was inversely proportional to his talent. Professor Koszul built in over 60 years of mathematical career, in the silence of his passions, an immense work, which makes him one of the great mathematicians of the XX's century, whose importance will only affirm with the time. In this troubled time and rapid transformation of society and science, the example of Professor Koszul must be regarded as a model for future generations, to avoid them the trap of fleeting glories and recognitions too fast acquired. The work of Professor Koszul is also a proof of fidelity to his masters and in the first place to Prof. Elie Cartan, who inspired him throughout his life. Henri Cartan writes on this subject "I do not forget the homage he paid to Elie Cartan's work in Differential Geometry during the celebration, in Bucharest, in 1969, of the centenary of his birth. It is not a coincidence that this centenary was also celebrated in Grenoble the same year. As always, Koszul spoke with the discretion and tact that we know him, and that we love so much at home".

We will conclude by quoting Jorge Luis Borges "both forgetfulness and memory are apt to be inventive" (Doctor Brodie's report). Our generation and previous one have forgotten or misunderstood the depth of the work of Jean-Louis Koszul and Elie Cartan on the study of bounded homogeneous domains. It is our responsibility to correct this omission, and to make it the new inspiration for the Geometric Science of Information. We will invite readers to listen to the last interview of Jean-Louis Koszul for 50th birthday of Joseph Fourier Institute

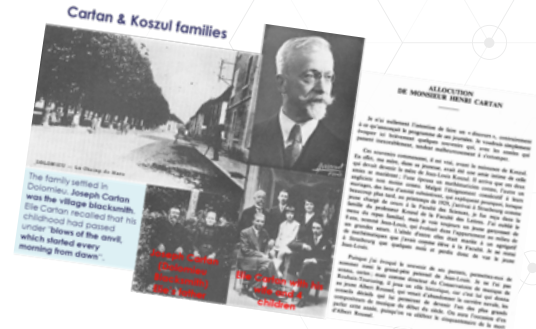


// Elie Cartan 150th birthday (1869 - 2019)

The mathematician Élie CARTAN (1869-1951) was born in Dolomieu. Son of a blacksmith, this young boy is noticed by the teacher of the communal school of Dolomieu. He appointed him to the attention of the cantonal delegate, Antonin DUBOST, who later became President of the Senate. A. DUBOST takes under his protection the young Elie and makes him study at the College of Vienne, then at the Lycee of Grenoble and finally at the Lycee Janson-de-Sailly in Paris. There, after a year of preparation, Élie CARTAN was received at the École Normale Supérieure in 1888. His doctoral thesis (1894), devoted to the classification of Lie groups, was a historic event. His later work on complex semi-simple groups, then real, had him appointed at the Sorbonne in 1909.

What made him famous all over the world was on the one hand his collaboration, after 1920, with the German mathematician Hermann WEYL, on the global study of Lie groups, on the other hand, the use he did the theory of groups of differential Geometry, then his theory of «generalized spaces» which was applicable in the theory of Relativity and which was the occasion of a long exchange of letters between Albert EINSTEIN and Elie CARTAN. These letters were published by Princeton University Press in 1979, on the occasion of the centenary of the birth of EINSTEIN. It is also to Elie CARTAN that we owe the systematic introduction of «differential exterior forms» in Differential Geometry.

Elie CARTAN was elected to the Academy of Sciences «in 1931. In 1969, the centenary of his birth was celebrated by international symposia in Bucharest and Grenoble, a street of which bears his name.



// Keynote speakers

Invited Honorary Speaker

**Gérard LETAC***(Université Paul Sabatier, Toulouse)**Classifying the exponential families by moving a convex function
Emeritus at the Université Paul Sabatier in mathematics*

Birth in 1940. Undergraduate and graduate studies Université de Caen, Agrégation in mathématiques in 1962. Doctorat d'Etat Clermont in 1972 (Thesis advisor Paul Malliavin). Professor at the Université Paul Sabatier from 1973 to 2003. Other positions as maître- assistant in Orsay from 1962 to 1966; associated professor, University of Montréal from 1966 to 1969; chairman of the computer science department at IUT de Clermont from 1969 to 1973. Supervision of 26 doctorate thesis, and publications of 130 papers.

Scientific interests:

- ▣ Harmonic analysis and probabilities: Markov chains on permutations ('Libraries'); Gelfand pairs and group actions on Markov chains, homogeneous trees; Dirichlet processes.
- ▣ Random matrices: Random walks on $SL(2;R)$ and random continuous fractions. Wishart and non central Wishart laws on symmetric cones.
- ▣ Theoretical statistics: Fisher information; exponential families and their variance functions; Gaussian and discrete graphical models.

Keynote Address: Classifying the exponential families by moving a convex function

This lecture is a tribute to the work made around 1990 by four graduate students: Marianne Mora, Muriel Casalis, Abdelhamid Hassairi and Célestin Kokonendji.

Draw the graph of a convex function k on R^d and make an affine transformation with positive determinant in R^{d+1} of such a surface. What you obtain is probably not the graph of a function anymore but a small proportion of it can be seen as the graph of a new convex function. When this process is applied to the convex function k_μ which is the log of the Laplace transform of a positive measure in R^d you may be linking two very different natural exponential

families. For instance for $d = 1$ you link in that way the Gaussian laws $N(m; 1)$ with the inverse Gaussian distributions. In many cases a probabilistic interpretation of the link can be provided. This is a genuine equivalence relation on the set of exponential families, and an equivalence class is called an orbit. Through a tool called the variance functions the above affine transformation can be read as a transformation of the variance function: the orbits of the quadratic real variance functions are due to M. Mora, M. Casalis has described the simple quadratic variance functions in R^d , A. Hassairi has found their orbits and C. Kokonendji has found the orbits of a rich class in R : One can also mention the recent study of the real variance functions $\sqrt{m^2+C}$ by elliptic functions and their orbits.

Keynote References

- ▣ M. Casalis, « Les $2n+4$ types de familles exponentielles naturelles sur R^n à fonction variance quadratique simple. », Comptes rendus de l'Académie des Sciences de Paris Série 1, 1994.
- ▣ A. Hassairi, « Les $d + 3$ G-orbites de la classe de Moris Mora des familles exponentielles de R^d », Comptes rendus de l'Académie des Sciences de Paris, Série 1, 1993.
- ▣ C. Kokonendji, "Exponential families with variance functions $P(\sqrt{\Delta})$ ", Test, Feb. 1994.
- ▣ G. Letac and M. Mora, "Natural exponential families with cubic variances", Ann. Statist. 18, 1-37. 1990
- ▣ G. Letac, "Associated exponential families and elliptic functions.", in The Fascination of Probability, Statistics and their Applications in honor of 'Ole Barndorff-Nielsen. Springer, New York, 53-84, 2015.

Guest Honorary Speaker



Karl Friston

(Wellcome Trust Centre for Neuroimaging)

Markov blankets and Bayesian mechanics MB, BS, MA, MRCPsych, FMedSci, FRSB, FRS

Wellcome Principal Fellow

Scientific Director: Wellcome Trust Centre for Neuroimaging

Institute of Neurology, UCL

Karl Friston is a theoretical neuroscientist and authority on brain imaging. He invented statistical parametric mapping (SPM), voxel-based morphometry (VBM) and dynamic causal modelling (DCM). These contributions were motivated by schizophrenia research and theoretical studies of value-learning, formulated as the dysconnection hypothesis of schizophrenia. Mathematical contributions include variational Laplacian procedures and generalized filtering for hierarchical Bayesian model inversion. Friston currently works on models of functional integration in the human brain and the principles that underlie neuronal interactions. His main contribution to theoretical neurobiology is a free-energy principle for action and perception (active inference). Friston received the first Young Investigators Award in Human Brain Mapping (1996) and was elected a Fellow of the Academy of Medical Sciences (1999). In 2000 he was President of the international Organization of Human Brain Mapping. In 2003 he was awarded the Minerva Golden Brain Award and was elected a Fellow of the Royal Society in 2006. In 2008 he received a Medal, College de France and an Honorary Doctorate from the University of York in 2011. He became of Fellow of the Royal Society of Biology in 2012, received the Weldon Memorial prize and Medal in 2013 for contributions to mathematical biology and was elected as a member of EMBO (excellence in the life sciences) in 2014 and the Academia Europaea in (2015). He was the 2016 recipient of the Charles Branch Award for unparalleled breakthroughs in Brain Research and the Glass Brain Award, a lifetime achievement award in the field of human brain mapping. He holds Honorary Doctorates from the University of Zurich and Radboud University.

Keynote Address: Markov blankets and Bayesian mechanics

This presentation offers a heuristic proof (and simulations of a primordial soup) suggesting that life—or biological self-organization—is an inevitable and emergent property of any (weakly mixing) random dynamical system that possesses a Markov blanket. This conclusion is based on the following arguments: if a system can be differentiated from its external milieu, heat bath or environment, then the system's internal and external states must be conditionally independent. These independencies induce a Markov blanket that separates internal and external states. This separation means that internal states will appear to minimize a free energy functional of blanket states – via a variational principle of stationary action. Crucially, this equips internal states with an information geometry, pertaining to probabilistic beliefs about something; namely external states. Interestingly, this free energy is the same quantity that is optimized in Bayesian inference and machine learning (where it is known as an evidence lower bound). In short, internal states (and their Markov blanket) will appear to model—and act on—their world to preserve their functional and structural integrity. This leads to a Bayesian mechanics, which can be neatly summarised as self-evidencing.

Keynote Speakers

**Elena Celledoni**

(Norwegian University of Science and Technology)

Structure preserving algorithms for geometric numerical integration

Professor at Department of Mathematical Sciences, Norwegian University of Science and Technology (NTNU), Trondheim, Norway

Elena Celledoni received her Master degree in mathematics from the University of Trieste in 1993, and her Ph.D in computational mathematics from the University of Padua, Italy, 1997. She held post doc positions at the University of Cambridge, UK, at the Mathematical Sciences Research Institute, Berkeley, California and at NTNU.

Her research field is in numerical analysis and in particular structure preserving algorithms for differential equations and geometric numerical integration.

Keynote Address: Structure preserving algorithms for geometric numerical integration

Computations of differential equations are of fundamental importance in applied mathematics. While historically the main quest was to derive all-purpose algorithms such as finite difference, finite volume and finite element methods for space discretization, Runge–Kutta and linear multistep methods for time integration, in the last 25 years the focus has shifted to special classes of differential equations and purpose-built algorithms that are tailored to preserve special features of each class. This has given rise to the new field of geometric numerical integration and structure preserving discretizations.

In this talk, we will give an introduction to structure preserving algorithms, focussing in particular on Lie group methods and on integrators on Riemannian manifolds. We consider a selection of applications of these methods. Local and global error bounds are derived in terms of the Riemannian distance function and the Levi-Civita connection. Finally, the notion of discrete gradient methods is generalised to Lie groups and to Riemannian manifolds. This approach leads to structure preserving methods for energy preserving as well as energy dissipative systems. We show that the methods are competitive with other approaches when applied to Riemannian gradient flow systems arising in diffusion tensor imaging.

Keynote References

- ▣ E Celledoni, S Eidnes, B Owren, T Ringholm, Energy preserving methods on Riemannian manifolds, arXiv preprint arXiv:1805.07578
- ▣ Elena Celledoni, Sølve Eidnes, Brynjulf Owren, and Torbjørn Ringholm, Dissipative Numerical Schemes on Riemannian Manifolds with Applications to Gradient Flows, *SIAM J. Sci. Comput.*, 40(6), A3789–A3806.
- ▣ E Celledoni, S Eidnes, A Schmeding, Proceedings of the Abel Symposium 2016, Springer, Shape analysis on homogeneous spaces: a generalised SRVT framework
- ▣ E Celledoni, M Eslitzbichler, A Schmeding, Shape Analysis on Lie Groups with Applications in Computer Animation, *Journal of Geometric Mechanics (JGM)*, 8 (3)
- ▣ Elena Celledoni, Håkon Marthinsen, Brynjulf Owren An introduction to Lie group integrators – basics, new developments and applications



Gabriel Peyré

(CNRS, Ecole Normale Supérieure)
Optimal Transport for Machine Learning

Gabriel Peyré is senior researcher at the Centre Nationale de Recherche Scientifique (CNRS) and professor at the Ecole Normale Supérieure, Paris. His research is focused on developing mathematical and numerical tools for imaging sciences and machine learning. He is the creator of the «Numerical tour of data sciences» (www.numerical-tours.com), a popular online repository of Python/Matlab/Julia/R resources to teach mathematical data sciences. His research was supported by a ERC starting grant (SIGMA-Vision, 2010-2015) and is now supported by a ERC consolidator grant (NORIA 2017-2021). He is the 2017 recipient of the Blaise-Pascal prize from the French Academy of sciences, awarded each year to a young applied mathematician.

Keynote address: Optimal Transport for Machine Learning

Optimal transport (OT) has become a fundamental mathematical tool at the interface between calculus of variations, partial differential equations and probability. It took however much more time for this notion to become mainstream in numerical applications. This situation is in large part due to the high computational cost of the underlying optimization problems. There is a recent wave of activity on the use of OT-related methods in fields as diverse as image processing, computer vision, computer graphics, statistical inference, machine learning. In this talk, I will review an emerging class of numerical approaches for the approximate resolution of OT-based optimization problems. This offers a new perspective for the application of OT in high dimension, to solve supervised (learning with transportation loss function) and unsupervised (generative network training) machine learning problems.

Keynote References

▣ Gabriel Peyré and Marco Cuturi, «Computational Optimal Transport», 2018, <https://optimaltransport.github.io/>



Jean-Baptiste Hiriart-Urruty

(Université de Toulouse)
Pierre de FERMAT (ca. 1605-1665): lawyer, philologist and illustrious mathematician ... but enigmatic

Jean-Baptiste Hiriart-Urruty is professor emeritus at the Université Paul Sabatier in Toulouse since 2015. It holds a PhD in mathematics from the Université Blaise Pascal in Clermont-Ferrand and an habilitation. He was full time professor in mathematics at University Paul Sabatier from 1981 to 2015. His research topics are variational calculus (convex, non smooth and applications) and optimization (global optimization, non smooth, non convex). He has also many contributions in the history of mathematics and mathematicians and in dissemination of mathematical science towards general public.

Keynote address: Pierre de FERMAT (ca. 1605-1665): lawyer, philologist and illustrious mathematician ... but enigmatic

Pierre de Fermat, born in Lomagne (one hour driving from Toulouse) at the beginning of the 17th century, shared his life between his professional responsibilities (as a lawyer in Toulouse and Castres), his taste for arts, humanities and languages, and his passion for mathematics, discipline that earned him his fame. From this historical context, we propose to divide our presentation into three parts.

1. The 17th century, period of “mathematization” of physical concepts such as velocity and acceleration. Contributions of Fermat, Pascal, Descartes (to the beginnings of Analytical Geometry, Probability Theory, modern Number Theory) ... and also of some “local” ones (from the Occitanie region).
2. The intellectual environment and the “forging” of science in the 17th century; a light on the work and the mathematical style of Fermat; an inevitable wink here on the properties of some integers ...
3. Fermat in the collective memory and news nowadays (names of streets, prices, books, movies, ...).

J.-B. Hiriart-Urruty contributed to the collective work “Pierre de FERMAT (ca. 1605-1665): lawyer, philologist and illustrious mathematician ... but enigmatic”, the most recent published book on the subject (Midi-Pyrénées Editions and Federal University of Toulouse, 2017; 18 euros). This book is among the winners of the 2017 Tangente Book Prize (1st accessit).

// Program

Sessions

▪ **Probability on Riemannian Manifolds***Marc Arnaudon, Ana Bela Cruzeiro*▪ **Optimization on Manifold***Nicolas Brunel, Fabrice Gamboa*▪ **Shape Space***Nicolas Charon, Pietro Gori*▪ **Statistics on non-linear data***Xavier Pennec, Thomas Hotz*▪ **Lie Group Machine Learning***Elena Celledoni, Frédéric Barbaresco*▪ **Statistical Manifold & Hessian Information Geometry***Michel Nguiffo Boyom, Hiroshi Matsuzoe*▪ **Monotone embedding and affine immersion of probability models***Jun Zhang, Atsumi Ohara*▪ **Non-parametric Information Geometry***Lorenz Schwachhöfer, Nigel Newton*▪ **Divergence Geometry***Frank Nielsen, Wolfgang Stummer*▪ **Computational Information Geometry***Frank Nielsen, Olivier Schwander*▪ **Wasserstein Information Geometry / Optimal Transport***Guido Montufar, Wuchen Li*▪ **Geometric structures in thermodynamics and statistical physics***Goffredo Chirco, François Gay-Balmaz*▪ **Geometric and structure preserving discretizations***François Gay-Balmaz, Joël Bensoam*▪ **Geometry of Quantum States***Florio Maria Ciaglia, Jan Naudts*▪ **Geometry of Tensor-Valued Data***Jesús Angulo, Geert Verdoolaege*▪ **Geometric Mechanics***Géry de Saxcé, Jean Lerbet*▪ **Geometric Science of Information Libraries***Nina Miolane, Alice Le Brigant*▪ **Poster Session***Pierre Baudot*

// Tuesday, August 27th

08:00-08:30

Registration desk – Badge withdrawing

08:30-09:00

Room: Amphi Bellonte**Opening Session:** *Frédéric Barbaresco, Frank Nielsen, Mathy Gonon*

09:00-10:00

Room: Amphi Bellonte**Keynote Speaker:** *Jean-Baptiste Hiriart-Urruty*
Pierre de FERMAT (ca. 1605-1665): lawyer, philologist and illustrious mathematician ... but enigmatic

10:00-10:30

Coffee break - **Room: Amphi Bellonte Hall**

10:30-12:30

Room: Amphi Bellonte**Session:** *SHAPE SPACE***Chairs:** *Nicolas Charon, Pietro Gori*▪ **On geometric properties of the textile set and strict textile set***Tomonari Sei and Ushio Tanaka*

Abstract: The textile plot is a tool for data visualization proposed by Kumasaka and Shibata (2008). The textile set is a geometric object aimed at understanding what the textile plot outputs. In this paper, we find additional facts on a proper subset called the strict textile set. Furthermore, we study differential and analytical geometric properties of the textile set.

▪ **Inexact elastic shape matching in the square root normal field framework***Nicolas Charon, Martin Bauer and Philipp Harms*

Abstract: This paper puts forth a new formulation and algorithm for the elastic matching problem on unparametrized curves and surfaces. Our approach combines the frameworks of square root normal fields and varifold fidelity metrics into a novel framework, which has several

potential advantages over previous works. First, our variational formulation allows us to minimize over reparametrizations without discretizing the reparametrization group. Second, the objective function and gradient are easy to implement and efficient to evaluate numerically. Third, the initial and target surface may have different samplings and even different topologies. Fourth, texture can be incorporated as additional information in the matching term similarly to the Fshape framework. We demonstrate the usefulness of this approach with several numerical examples of curves and surfaces.

▪ **Signatures in Shape Analysis: an Efficient Approach to Motion Identification***Elena Celledoni, Pål Erik Lystad and Nikolas Tapia*

Abstract: Signatures provide a succinct description of certain features of paths in a reparametrization invariant way. We propose a method for classifying shapes based on signatures, and compare it to current approaches based on the SRV transform and dynamic programming.

▪ **Dilation operator approach for time/Doppler spectra characterization on $SU(n)$** *Guillaume Bouleux and Frederic Barbaresco*

Abstract: We propose in this work the use of Dilation theory for non-stationary signals and their time/Doppler spectra to embed the underlying spectral measure on the Special Unitary group $SU(n)$. The Dilation theory gives access to rotation-like matrices built in with partial correlation coefficients. Due to the non-stationary condition, the time/Doppler spectra is associated with a path on $SU(n)$. We use next the Square root Velocity Transform which has been proven to be equivalent to a first order Sobolev metric on the space of shapes. Because the metric in the space of curve naturally extends to the space of shapes, this enables a comparison between curves' shapes and allows then the classification of time/Doppler spectra.

▪ **Selective metamorphosis for growth modelling with applications to landmarks***Andreas Bock, Alexis Arnaudon and Colin Cotter*

Abstract: We present a framework for shape matching in computational anatomy allowing users control of the degree to which the matching is diffeomorphic. The control is a function defined over the image describing where to violate the diffeomorphic constraint. The location can either be specified from prior knowledge of the growth location or learned from data. We consider landmark matching

and infer the distribution of a finite dimensional parameterisation of the control via Markov chain Monte Carlo. Preliminary analytical and numerical results are shown and future paths of investigation are laid out.

Room: G11

Session: GEOMETRIC MECHANICS

Chairs: *Géry de Saxcé, Jean Lerbet*

▪ Intrinsic Incremental Mechanics

Jean Lerbet

Abstract: We produce a coordinate free presentation of some concepts usually involved in incremental mechanics (tangent linear stiffness matrix, stability, load paths for example) but not always well founded. Thanks to the geometric language of vector bundles, a well defined geometrical object may be associated to each of these tools that allows us to understand some latent difficulties linked with these tools due to the absence of a natural connection and also to extend some of our results in linear stability to a non linear framework.

▪ Multi-symplectic Extension of Lie Group Thermodynamics for Covariant Field Theories

Goffredo Chirco, Marco Laudato and Fabio M. Mele

Abstract: We propose a spacetime covariant generalisation of Souriau's Lie group thermodynamics in the multi-symplectic framework for relativistic field theories. Starting from a covariant Gibbs state functional, defined with respect to the lifted action of the diffeomorphisms group on the extended phase space of the fields, we elaborate a statistical mechanic description of fully constrained first order field theories.

▪ Euler-Poincaré equation for Lie groups with non null symplectic cohomology. Application to the Mechanics

Gery De Saxce

Abstract: Let G be a symplectic group on a symplectic manifold N . To any momentum map $\mu: N \rightarrow \mathfrak{g}^*$, one can associate a class of symplectic cohomology cocs. It does not depend on the choice of the momentum map but only on the structure of the Lie group G . It forwards an affine left action $\mu = a \cdot \mu' = \text{Ad}^*(a) \mu' + \text{cocs}(a)$ of G on \mathfrak{g}^* .

If G is a Lie subgroup of the affine group, one can define an associated affine connection as a field of \mathfrak{g} -valued 1-forms on a G -principal

bundle of affine frames $\pi: F \rightarrow M$. Let ω be a smooth field of 2-form on $\mathfrak{g}^* \times F$ defined by: $\omega = \langle \mu, f \rangle$

On each orbit μ , ω is the pull-back of Kirillov-Kostant-Souriau symplectic form by the projection $\mu: \mathfrak{g}^* \rightarrow \mathfrak{g}^*$. The G -principal bundle $\mathfrak{g}^* \times F$ is a presymplectic bundle of symplectic form and μ is a momentum map.

The equation of motion $d\langle \mu, f \rangle \in \text{Ker}(\omega)$ expresses the fact that the momentum is parallel-transported. It generalizes Euler-Poincaré equation when the class of symplectic cohomology of the group is not null, especially for the important case of Galileo's group. Mitsuhiro Itoh and Hiroyasu Satoh: Riemannian distance and diameter of the space of probability measures and the parametrix. Information geometry of the space $P(M)$ of probability measures, defined on a compact smooth Riemannian manifold M and equipped with Fisher metric G , is investigated from the viewpoint of Riemannian geometry. The function $\pi: P(M) \times P(M) \rightarrow [0, \infty)$ associated to the geometric mean of two probability measures is introduced. From the formulae of Levi-Civita geodesics the Riemannian distance $d(\mu, \nu)$ of the infinite dim. manifold $(P(M), G)$ is exactly given by the function π . By applying the parametrix of the heat kernel of M it is shown that the diameter of $(P(M), G)$ equals π .

▪ Geometric numerical methods for mechanics

Joël Bensoam and Pierre Carré

Abstract: Due to the increasing demands for modeling large-scale and complex systems, designing optimal controls, and conducting optimization tasks, many real-world applications require sophisticated models. Geometric methods are designed to capture the underlying structure of the system at hand and to preserve the global qualitative or geometric properties of the flow, such as symplecticity, volume preservation and symmetry. A survey on three of such structure preserving numerical methods is proposed in the present article. Testing the validity of such simulations is achieved by exhibiting analytically solvable models and comparing the result of simulations with their exact behavior.

▪ Souriau Exponential Map Algorithm for Machine Learning on Matrix Lie Groups

Frédéric Barbaresco

Abstract: Jean-Marie Souriau extended Urbain Jean Joseph Leverrier algorithm to compute characteristic polynomial of a matrix in 1948. This Souriau algorithm could be used to compute exponential

map of a matrix that is a challenge in Lie Group Machine Learning. Main property of Souriau Exponential Map numerical scheme is its scalability by highly parallelization. Main drawback of most efficient classical algorithm based on Krylov iterates cannot be parallelized. Souriau algorithm has a complexity $O(n^4)$ in sequential computation, and so cannot compete with Krylov-based algorithm, but Souriau algorithm has been parallelized by L. Csanky, proving that characteristic polynomial computation could be solved in parallel time $\log_2 n$ with a polynomial number of processors. Souriau algorithm parallelization by Czansky has been improved more recently by Preparata & Sarwate using fast matrix product, and by Keller-Gehrig using matrix reduction.

Room: G13

Session: GEOMETRY OF TENSOR-VALUED DATA

Chairs: Jesús Angulo, Geert Verdoolaege

▪ R-Complex Finsler Information Geometry Applied to Manifolds of Systems

Christophe Corbier

Abstract: In this article information geometry is applied to the models of linear discrete time-invariant systems MLTI, S with external input and zeros outside the unit disc of the z-plane. A new information geometry of manifolds of systems based R-Complex Finsler spaces with three metric tensors is presented. First, two metric tensors in Hermitian spaces H where $H-1$ corresponds to the zeros outside the unit disc (exogenous zeros) and $H+1$ to the zeros and poles inside the unit disc (endogenous zeros-poles). Then, one metric tensor as a mixing of exogenous zeros and endogenous zeros-poles in a non-Hermitian space H . Experimental results are presented from a semi-finite acoustic waves guide.

▪ Minkowski Sum of Ellipsoids and Means of Covariance Matrices

Jesús Angulo

Abstract: The Minkowski sum and difference of two ellipsoidal sets are in general not ellipsoidal. However, in many applications, it is required to compute the ellipsoidal set which approximates the Minkowski operations in a certain sense. In this study, an approach based on the so-called ellipsoidal calculus, which provides parameterized families of external and internal ellipsoids that tightly approximate the Minkowski sum and difference of ellipsoids, is considered. Approximations are tight along a direction l in the sense that the support functions on l of the ellipsoids are equal to the support function on l of the sum and difference. External (resp.

internal) support function-based approximation can be then selected according to minimal (resp. maximal) measures of volume or trace of the corresponding ellipsoid. The connection between the volume-based approximations to the Minkowski sum and difference of two positive definite matrices and their mean using their Euclidean or Riemannian geometries is developed, which is also related to their Bures-Wasserstein mean.

▪ Hyperquaternions: An Efficient Mathematical Formalism for Geometry

Patrick R Girard, Patrick Clarysse, Romaric Pujol, Robert Goutte and Philippe Delachartre

Abstract: Hyperquaternions being defined as a tensor product of quaternion algebras (or a subalgebra thereof), they constitute Clifford algebras due to a theorem by Clifford [1, 5]. Examples of hyperquaternions are quaternions H ($e_1=i, e_2=j$), biquaternions $H C$ ($e_1=i, e_2=j, e_3=kl$), tetraquaternions $H H$ ($e_1=j, e_2=k, e_3=kl$), and so on $H H C, \dots$. Independently of Clifford, Lipschitz [6] discovered what is now known as the even subalgebra and gave the formula of n dimensional rotations in euclidean spaces $y = axa^{-1}$ ($a \in C_n$). Moore [7], working on a canonical decomposition of Euclidean rotations, was to call the elements of Lipschitz's algebra hyperquaternions which justifies the above terminology. Hyperquaternions provide an efficient algebraic framework for geometry, endowed with an associative exterior product [2]. The paper presents a hyperquaternion formulation of pseudo-euclidean rotations and the Poincaré group in n dimensions (via dual hyperquaternions). It also develops, as an extension of Moore's method, a hyperquaternion canonical planar decomposition of these groups in pseudo-euclidean spaces. Potential applications include in particular, moving frames, robotics and machine learning [8].

References:

- [1] Girard, Patrick R. et al., Hyperquaternions: A New Tool for Physics. Adv. Appl. Clifford Algebras (2018) 28:68
- [2] Girard, Patrick R. et al., Differential Geometry Revisited by Biquaternion Clifford Algebra. In J.-D. Boissonat et al (Eds.): Curves and Surfaces (Springer, 2015).
- [3] Girard, Patrick R., Quaternions, Clifford Algebras and Relativistic Physics (Birkhauser, Basel, 2007).
- [4] Girard, Patrick R., Quaternions, Algèbre de Clifford et Physique relativiste (PPUR, Lausanne, 2004).
- [5] Clifford, W. K., Applications of Grassmann's extensive algebra, Amer. J. Math., 1 (1878), pp. 350-358.
- [6] Lipschitz R., Principes d'un calcul algébrique qui contient comme

espèces particulières le calcul des quantités imaginaires et des quaternions, C. R. Acad. Sci. Paris, 91 (1880), pp. 619–621, 660–664.

[7] Moore, C. J. E., Hyperquaternions, Journal of Mathematical Physics, 1 (1922), pp. 63–77.

[8] Xuanyu Zhu, Yi Xu, Hongteng Xu, and Changjian Chen. Quaternion Convolutional Neural Networks. ECCV, 2018.

▪ Alpha-power sums on symmetric cones

Keiko Uohashi

Abstract: In this paper, we define α -power sums of two or more elements on symmetric cones. For two elements, α -power sums, which are generalized parallel sums, are defined on our previous paper. We mention interpolation for α -power sums, which is not defined on our previous paper. It is shown that the synthesized resistances of α -series parallel circuits naturally correspond to α -power sums. We also mention relations with power sums and arithmetic, geometric, harmonic and α -power means, where α is a parameter of dualistic structure on information geometry.

▪ Packing Bounds for Outer Products with Applications to Compressive Sensing

Sebastian Semper and Thomas Hotz

Abstract: In order to obtain good reconstruction guarantees for typical compressive sensing scenarios, we translate the search for good compression matrices into a ball packing problem in a suitable projective space. We then derive such reconstruction guarantees for two relevant scenarios, one where the matrices are unstructured and one where they have to be Khatri-Rao products. Finally, we demonstrate how the proposed method can be implemented with a physically motivated numerical optimization scheme, and how it compares to a conventional scheme of random compression matrices.

12:30–13:30

 Lunch break

13:30–15:30

Session: LIE GROUP MACHINE LEARNING

Chairs: *Elena Celledoni, Frédéric Barbaresco*

Room: **Amphi Bellonte**

▪ On a method to construct exponential families by representation theory

Koichi Tojo and Taro Yoshino

Abstract: Exponential family plays an important role in information geometry. In [TY18], we introduced a method to construct an exponential family $\mathcal{P} = \{p\}$ on a homogeneous space G/H from a pair (V, v_0) . Here V is a representation of G and v_0 is an H -fixed vector in V . Then the following questions naturally arise: (Q1) when is the correspondence p injective? (Q2) when do distinct pairs (V, v_0) and (V', v_0') generate the same family? In this paper, we answer these two questions (Theorems 1 and 2). Moreover, in Section 3, we consider the case $(G, H) = (R^{>0}, \{1\})$ with certain representation on R^2 . Then we see the family obtained by our method is essentially $GI(G)$ (Generalized inverse Gaussian distribution).

▪ Lie Group Machine Learning & Gibbs Density on Poincaré Unit Disk from Souriau Lie Groups Thermodynamics and $SU(1,1)$ Coadjoint Orbits

Frederic Barbaresco

Abstract: In 1969, Jean-Marie Souriau has introduced a “Lie Groups Thermodynamics” in Statistical Mechanics in the framework of Geometric Mechanics. This Souriau’s model considers the statistical mechanics of dynamic systems in their «space of evolution» associated to a symplectic manifold by a Lagrange 2-form, and defines thanks to cohomology (non equivariance of the coadjoint operator on the moment map with appearance of a cocycle) a Gibbs density (of maximum entropy) that is covariant under the action of dynamic groups of physics (eg, Galileo’s group in classical physics). Souriau model is more general if we consider another Souriau theorem, that we can associate to a Lie group, an homogeneous symplectic manifold with a KKS 2-form on their coadjoint orbits. Souriau method could then be applied on Lie Groups to define a covariant maximum entropy density by Kirillov representation theory. We will illustrate this method for homogeneous Siegel domains and more especially for Poincaré unit disk by considering $SU(1,1)$ group coadjoint orbit and by computing its Souriau’s moment map. For this case, the coadjoint action on moment map is equivariant.

▪ Irreversible Langevin MCMC on Lie Groups

Alexis Arnaudon, Alessandro Barp and So Takao

Abstract: It is well-known that irreversible MCMC algorithms converge faster to its stationary distribution than reversible ones. Using the special geometric structure of Lie groups G , we construct an irreversible HMC-like MCMC algorithm on G , where we first update the momentum by solving an OU process on the corresponding Lie algebra \mathfrak{g} , and then running the Hamiltonian system on $G \times \mathfrak{g}$, followed by a Metropolis-Hastings correction step. In particular, when the OU process is simulated over sufficiently long times, we recover HMC as a special case. We illustrate this algorithm numerically using the example $G=SO(3)$.

▪ Predicting Bending Moments with Machine Learning

Henrik Sperre Sundklakk, Elena Celledoni, Nikita Kopylov and Halvor S. Gustad

Abstract: We investigate the possibility of predicting the bending moment of slender structures based on a limited number of deflection measurements. These predictions can help to estimate the wear and tear of the structures. We compare linear regression and a recurrent neural network on numerically simulated Euler-Bernoulli beam and drilling riser.

▪ The exponential of nilpotent supergroups in the theory of Harish-Chandra representations

Rita Fiorese, Claudio Carmeli and Veeravalli S. Varadarajan

Abstract: In this paper we discuss the exponential map in the case of nilpotent superalgebras. This provides global coordinates for nilpotent analytic supergroups, which are useful in the applications.

15:30-16:00

 Coffee break - **Room: Amphi Bellonte Hall**

16:00-18:00

Room: Amphi Bellonte

Session: GEOMETRIC STRUCTURES IN THERMODYNAMICS AND STATISTICAL PHYSICS

Chairs: Goffredo Chirco, François Gay-Balmaz

▪ Dirac structures in open thermodynamics

Hiroaki Yoshimura and Francois Gay-Balmaz

Abstract: Dirac structures are geometric objects that generalize Poisson structures and presymplectic structures on manifolds. They naturally appear in the formulation of constrained mechanical systems and play an essential role in the understanding of the interrelations between system elements in implicit dynamical systems. In this paper, we show how nonequilibrium thermodynamic systems can be naturally understood in the context of Dirac structures, by mainly focusing on the case of open systems, i.e., thermodynamic systems exchanging heat and matter with the exterior.

▪ From variational to single and double bracket formulations in nonequilibrium thermodynamics of simple systems

François Gay-Balmaz and Hiroaki Yoshimura

Abstract: A variational formulation for nonequilibrium thermodynamics was recently proposed in Gay-Balmaz and Yoshimura [2017a,b] for both discrete and continuum systems. This formulation extends the Hamilton principle of classical mechanics to include irreversible processes. In this paper, we show that this variational formulation yields a constructive and systematic way to derive from a unified perspective several bracket formulations for nonequilibrium thermodynamics proposed earlier in the literature, such as the single generator bracket and the double generator bracket. In the case of a linear relation between the thermodynamic fluxes and the thermodynamic forces, the metriplectic or GENERIC brackets are recovered. A similar development has been presented for continuum systems in Eldred and Gay-Balmaz [2019] and applied to multicomponent fluids

▪ A Homological Approach to Belief Propagation and Bethe Approximations

Olivier Peltre

Abstract: We introduce a differential complex of local observables given a decomposition of a global set of random variables into subsets. Its boundary operator allows us to define a transport equation equivalent to Belief Propagation. This definition reveals a set of conserved quantities under Belief Propagation and gives new insight on the relationship of its equilibria with the critical points of Bethe free energy.

- **About some systems-theoretic properties of Port Thermodynamic systems**

Arjan van der Schaft and Bernhard Maschke

Abstract: Recently a class of Hamiltonian control systems has been defined for open irreversible thermodynamic systems. These systems are Hamiltonian control systems defined on a symplectic manifold, however departing from standard Hamiltonian control systems, due to the property that the Hamiltonian function is homogeneous in the generalized momentum variables. In this paper we study the system-theoretic properties of these systems in particular the feedback equivalence and the passivity properties.

- **Expectation variables on a para-contact metric manifold exactly derived from master equations**

Shin-Itiro Goto and Hideitsu Hino

Abstract: Based on information and para-contact metric geometries, in this paper a class of dynamical systems is formulated for describing time-development of expectation variables. Here such systems for expectation variables are exactly derived from continuous-time master equations describing nonequilibrium processes.

Room: G11

Session: MONOTONE EMBEDDING AND AFFINE IMMERSION OF PROBABILITY MODELS

Chairs: Jung Zhang, Atsumi Ohara

- **Doubly autoparallel structure and its applications**

Atsumi Ohara

Abstract: In a statistical manifold, we can naturally define submanifolds that are simultaneously autoparallel with respect to both the primal and the dual affine connections of the statistical manifold. We call them doubly autoparallel submanifolds. The aim of this paper is to mainly introduce doubly autoparallelism on positive definite matrices in linear algebraic way and show its applicability to two related topics.

- **Toeplitz Hermitian Positive Definite Matrix Machine Learning based on Fisher Metric**

Yann Cabanes, Frederic Barbaresco, Marc Arnaudon and Jérémie Bigot

Abstract: Here we propose a method to classify radar clutter from radar data using an unsupervised classification algorithm. The data will be represented by Positive Definite Hermitian Toeplitz matrices and clustered using the Fisher metric. Once the clustering algorithm dispose of a large radar database, new radars will be able to use the

experience of other radars, which will improve their performances: learning radar clutter can be used to fix some false alarm rate created by strong echoes coming from hail, rain, waves, mountains, cities; it will also improve the detectability of slow moving targets, like drones, which can be hidden in the clutter, flying close to the landform.

- **Deformed exponential and the behavior of the normalizing function**

Francisca Leidmar Josué Vieira, Rui Facundo Vigelis, Luiza Helena Félix de Andrade and Charles Casimiro Cavalcante

Abstract: In this paper we consider the statistical manifold given from the deformed exponential φ . For the non-atomic case we establish a relation between the behavior of the deformed exponential function and the Δ_2 -condition and analyze the compartment of the normalizing function near to the boundary of its domain. In the purely atomic case we find an equivalent condition to the behavior that characterizes the deformed exponential discussed in this work. Moreover we prove a consequence from the fact the Musielak-Orlicz function does not satisfy the Δ_2 -condition.

- **Normalization problems for deformed exponential families**

Hiroshi Matsuzoe, Antonio M. Scarfone and Tatsuaki Wada

Abstract: In this study, after reviewing fundamental properties of ordinary exponential families, we consider geometry of deformed exponential families. We redefine a deformed logarithm function and a deformed exponential function. In fact, by adjusting the initial condition of deformed logarithm, we find that the α -representations in information geometry and the rectified linear unit (ReLU) belong the class of q -exponentials. Under the new definition of deformed exponential function, we discuss dually at structures for deformed exponential families. We also consider normalization problems for those families using the ReLU.

- **New Geometry of Parametric Statistical Models**

Jun Zhang and Gabriel Khan

Abstract: We provide an alternative differential geometric framework of the manifold M of parametric statistical models. While adopting the Fisher-Rao metric as the Riemannian metric g on M , we treat the original parameterization of the statistical model as affine coordinate chart on the manifold endowed with a flat connection, instead of using a pair of torsion-free affine connections with generally non-vanishing curvature. We then construct its g -conjugate connection which, while necessarily curvature-free, carries torsion in general. So instead of associating a statistical structure to M , we construct a statistical manifold admitting torsion (SMAT). We show that that M is dually flat if and only if torsion of the conjugate connection vanishes.

Room: G13

Session: DIVERGENCE GEOMETRY

Chairs: Frank Nielsen, Wolfgang Stummer

▪ **The Bregman chord divergence**

Frank Nielsen and Richard Nock

Abstract: Distances are fundamental primitives whose choice significantly impacts the performances of algorithms in signal processing. However selecting the most appropriate distance for a given task is an endeavor. Instead of testing one by one the entries of an ever-expanding dictionary of ad hoc distances, one rather prefers to consider parametric classes of distances that are exhaustively characterized by axioms derived from first principles. Bregman divergences are such a class.

However finely tuning a Bregman divergence is delicate since it requires to smoothly adjust a functional generator. In this work, we propose an extension of Bregman divergences called the Bregman chord divergences. This new class of distances does not require gradient calculations, uses two scalar parameters that can be easily tailored in applications, and generalizes asymptotically Bregman divergences.

▪ **Testing the number and the nature of the components in a mixture distribution**

Michel Broniatowski, Emilie Miranda and Wolfgang Stummer

Abstract: In this paper, we propose a test procedure for the number of components of mixture distribution in a parametric setting. The test statistic is based on divergence estimators derived through the dual form of the divergence in parametric models. We provide a standard limit distribution for the test statistic under the null hypothesis, that holds for mixtures of any number of components $k \geq 2$.

▪ **Robust estimation by means of scaled Bregman power distances. Part I: Nonhomogeneous data**

Wolfgang Stummer and Birgit Roensch

Abstract: In contemporary data analytics, one often models uncertainty-prone data as samples stemming from a sequence of independent random variables whose distributions are non-identical but linked by a common (scalar or multidimensional) parameter. For such a context, we present in the current Part I a new robustness-featured parameter-estimation framework, in terms of minimization of the scaled Bregman power distances of Stummer & Vajda [22] (see also [21]); this leads to a wide range of outlier-robust

alternatives to the omnipresent (non-robust) method of maximum-likelihood-examination, and extends the corresponding method of Ghosh & Basu [7]. In Part II (see [20]), we provide some applications of our framework to data from potentially rare but dangerous events described by approximate extreme value distributions.

This is an invited talk.

▪ **Robust estimation by means of scaled Bregman power distances. Part II: Extreme values.**

Birgit Roensch and Wolfgang Stummer

Abstract: In the separate Part I (see [23]), we have derived a new robustness-featured parameter-estimation framework, in terms of minimization of the scaled Bregman power distances of Stummer & Vajda [25] (see also [24]); this leads to a wide range of outlier-robust alternatives to the omnipresent non-robust method of maximum-likelihood-examination. In the current Part II, we provide some applications of our framework to data from potentially rare but dangerous events (modeled with approximate extreme value distributions), by estimating the correspondingly characterizing extreme value index (reciprocal of tail index); as a special subcase, we recover the method of Ghosh [9] which is essentially a robustification of the procedure of Matthys & Beirlant [19]. Some simulation studies demonstrate the potential partial superiority of our method.

This is an invited talk.

18:00-19:00

Keynote Speaker: Gérard Letac

Classifying the exponential families by moving a convex function

Room: Amphi Bellonte

19:00-20:00

 **Welcome reception : Nicolas Cazalis**

Room: Amphi Bellonte Hall

// Wednesday, August 28th

08:30-09:00

Registration desk – Badge withdrawing

09:00-10:00

Room: Amphi Bellonte**Keynote Speaker: Karl Friston****Markov blankets and Bayesian mechanics**

10:00-10:30

Coffee break - **Room: Amphi Bellonte Hall**

10:30-12:30

Room: Amphi Bellonte**Session: COMPUTATIONAL INFORMATION GEOMETRY****Chairs: Frank Nielsen, Olivier Schwander****▪ Topological methods for unsupervised learning***Leland McInnes*

Abstract: Unsupervised learning is a broad topic in machine learning with many diverse sub-disciplines. Within the field of unsupervised learning there are three major topics: dimension reduction; clustering; and anomaly detection. We seek to use the languages of topology and category theory to provide a unified mathematical approach to these three major problems in unsupervised learning.

▪ Geometry and fixed-rate quantization in Riemannian metric spaces induced by separable Bregman divergences*Erika Gomes-Goncalves, Henryk Gzyl and Frank Nielsen*

Abstract: Dual separable Bregman divergences induce dual Riemannian metric spaces which are isometric to the Euclidean space after non-linear monotone embeddings. We investigate fixed-rate quantization and the induced Voronoi diagrams in those metric spaces.

▪ The statistical Minkowski distances: Closed-form formula for Gaussian Mixture Models*Frank Nielsen*

Abstract: The traditional Minkowski distances are induced by the corresponding Minkowski norms in real-valued vector spaces. In this work, we propose statistical symmetric distances based on the Minkowski's inequality for probability densities belonging to Lebesgue spaces. These statistical Minkowski distances admit closed-form formula for Gaussian mixture models when parameterized by integer exponents.

▪ Parameter estimation with generalized empirical localization*Takashi Takenouchi*

Abstract: It is often difficult to estimate parameters of discrete models because of the computational cost for calculation of normalization constant, which enforces the model to be probability. In this paper, we consider a computationally feasible estimator for discrete probabilistic models using a concept of generalized empirical localization, which corresponds to the generalized mean of distributions and homogeneous γ -divergence. The proposed estimator does not require the calculation of the normalization constant and is asymptotically efficient.

▪ Properties of the Cross Entropy between ARMA Processes*Eric Grivel*

Abstract: In this paper, we propose to analyze the properties of the cross entropy between the probability density functions of k consecutive samples of wide-sense stationary ARMA processes. It is shown that, when k increases, the cross entropy has a transient behavior before tending to an affine function of k whose slope is defined by the so-called asymptotic increment. The latter depends on the parameters of the processes and can be a reasonable choice to characterize the cross entropy. Some illustrations are then given.

Room: G11**Session: STATISTICAL MANIFOLD & HESSIAN INFORMATION GEOMETRY****Chairs: Michel Nguiffo Boyom, Hiroshi Matsuzoe****▪ Inequalities for Statistical Submanifolds in Hessian Manifolds of Constant Hessian Curvature***Aliya Siddiqui, Kamran Ahmad and Cenap Ozel*

Abstract: In the present paper, we study Hessian and Einstein-Hessian manifolds with some examples. We establish optimizations of the intrinsic invariant (normalized scalar curvature) for a new extrinsic invariant (generalized normalized Casorati curvatures) on statistical submanifolds in a Hessian manifold of constant Hessian curvature by using algebraic technique. We consider the equality case of the derived inequalities.

▪ B. Y. Chen inequalities for statistical submanifolds in Sasakian statistical manifolds

Mohd Aquib, Michel Nguiffo Boyom, Ali H. Alkhaldi and Mohammad Hasan Shahid

Abstract: In this paper, we derive a statistical version of B. Y. Chen inequality for statistical submanifolds in the Sasakian statistical manifolds with constant curvature and discuss the equality case of the inequality. We also give some applications of the inequalities obtained.

▪ Generalized Wintgen inequality for Legendrian Submanifolds in Sasakian statistical manifolds

Michel Nguiffo Boyom, Zamrooda Jabeen, Mehraj Ahmad Lone, Mohamd Saleem Lone and Mohammad Hasan Shahid

Abstract: In 1999, Smet et.al. conjectured the generalized Wintgen inequality for submanifolds in real space forms. The commonly name used for this conjecture is DDVV conjecture proved independently by Ge and Tang(2008) and Lu(2011). Mihai Proved the Wintgen inequality for lagrangian and Legendrian submanifolds in complex space forms and Sasakian space forms in 2014 and 2017 respectively. In the present paper, we proved the same inequality for Legendrian submanifolds of Sasakian statistical manifolds.

▪ Logarithmic divergences: geometry and interpretation of curvature

Ting Kam Leonard Wong and Jiaowen Yang

Abstract: We study the logarithmic $L(\pm)$ -divergences which extrapolate the Bregman divergence and correspond to solutions to novel optimal transport problems. We show that the logarithmic divergence is equivalent to a conformal transformation of the Bregman divergence, and, via an explicit affine immersion, is equivalent to Kurose's canonical divergence. In particular, the $L(\pm)$ -divergence is a canonical divergence on a statistical manifold with constant sectional curvature. For such a manifold, we give a geometric interpretation of its sectional curvature in terms of how the divergence between a pair of primal and dual geodesics differ from the dually flat case. For expositional simplicity we only state results for the $L(-)$ -divergence.

▪ Hessian Curvature and Optimal Transport

Gabriel Khan and Jun Zhang

Abstract: Let X and Y be open subsets of Euclidean space equipped with probability measures μ and ν . We consider the problem of transporting μ to ν in way that minimizes a cost $c: X \times Y \rightarrow \mathbb{R}$ of the form $c(x,y) = \phi(x-y)$ or some convex function ϕ . For costs of this form, we introduce a new complex geometric interpretation of the optimal transport problem by considering an induced Sasaki metric on the tangent bundle of the set $X-Y$. In this framework, the MTW tensor is proportional to the orthogonal holomorphic bisectional curvature. This provides a geometric framework for optimal transport complimentary to the pseudo-Riemannian approach of Kim and McCann. We provide applications of this result to complex geometry, optimal transport and information geometry. This work expands upon previous work of the authors, which studied this problem in the context of log-divergences, which are relevant in information geometry and economics.

Room: G13

Session: *NON-PARAMETRIC INFORMATION GEOMETRY*

Chairs: *Lorenz Schwachhöfer, Nigel Newton*

▪ Divergence functions in Information Geometry

Domenico Felice and Nihat Ay

Abstract: A recently introduced canonical divergence D for a dual structure (g, \star) on a smooth manifold M in connection to other divergence functions. Finally, general properties of D are outlined and critically discussed.

▪ Sobolev Statistical Manifolds and Exponential Models

Nigel Newton

Abstract: This paper develops Sobolev variants of the non-parametric statistical manifolds appearing in [8] and [9]. The manifolds are modelled on a particular class of weighted, mixed-norm Sobolev spaces, including a Hilbert-Sobolev space. Densities are expressed in terms of a deformed exponential function having linear growth, which lifts to a continuous nonlinear superposition (Nemytskii) operator. This property is used in the construction of finite-dimensional mixture and exponential submanifolds, on which approximations can be based. The manifolds of probability measures are developed in their natural setting, as embedded submanifolds of those of finite measures.

- **Minimization of the Kullback-Leibler divergence over a log-normal exponential arc**

Paola Siri and Barbara Trivellato

Abstract: The Kullback-Leibler divergence of a given log-normal density from a log-normal exponential arc is minimized, obtaining an optimal value which is robust with respect to homogeneous transformations which leave the correlation of the random variables involved unchanged.

- **Riemannian distance and diameter of the space of probability measures and the parametrix**

Mitsuhiro Itoh and Hiroyasu Satoh.

Abstract: Mitsuhiro Itoh and Hiroyasu Satoh: Riemannian distance and diameter of the space of probability measures and the parametrix. Information geometry of the space $P(M)$ of probability measures, defined on a compact smooth Riemannian manifold M and equipped with Fisher metric G , is investigated from the viewpoint of Riemannian geometry. The function $\pi: P(M) \times P(M) \rightarrow [0, \infty)$ associated to the geometric mean of two probability measures is introduced. From the formulae of Levi-Civita geodesics the Riemannian distance $d(\cdot, \cdot)$ of the infinite dim. manifold $(P(M), G)$ is exactly given by the function π . By applying the parametrix of the heat kernel of M it is shown that the diameter of $(P(M), G)$ equals π .

12:30-13:30



Lunch break

13:30-15:10

Room: Amphi Bellonte

Session: STATISTICS ON NON-LINEAR DATA

Chairs: Xavier Pennec, Thomas Hotz

- **A unified formulation for the Bures-Wasserstein and Log-Euclidean/Log-Hilbert-Schmidt distances between positive definite operators**

Minh Ha Quang

Abstract: This work presents a parametrized family of distances, namely the Alpha Procrustes distances, on the set of symmetric, positive definite (SPD) matrices. The Alpha Procrustes distances provide a unified formulation encompassing both the Bures-

Wasserstein and Log-Euclidean distances between SPD matrices. This formulation is then generalized to the set of positive definite Hilbert-Schmidt operators on a Hilbert space, unifying the infinite-dimensional Bures-Wasserstein and Log-Hilbert-Schmidt distances. The presented formulations are new both in the finite and infinite-dimensional settings.

- **Exploration of Balanced Metrics on Symmetric Positive Definite Matrices**

Yann Thanwerdas and Xavier Pennec

Abstract: Symmetric Positive Definite (SPD) matrices have been used in many fields of medical data analysis. Many Riemannian metrics have been defined on this manifold but the choice of the Riemannian structure lacks a set of principles that could lead one to choose properly the metric.

This drives us to introduce the principle of balanced metrics that relate the affine-invariant metric with the Euclidean and inverse-Euclidean metric, or the Bogoliubov-Kubo-Mori metric with the Euclidean and log-Euclidean metrics. We introduce two new families of balanced metrics, the mixed-power-Euclidean and the mixed-power-affine metrics and we discuss the relation between this new principle of balanced metrics and the concept of dual connections in information geometry.

- **Affine-invariant midrange statistics**

Cyrus Mostajeran, Christian Grussler and Rodolphe Sepulchre

Abstract: We formulate and discuss the affine-invariant matrix midrange problem on the cone of positive definite Hermitian matrices $P(n)$ of dimension n that is based on the Thompson metric. A particular computationally efficient midpoint of this metric is investigated as a highly scalable candidate for an average of two positive definite matrices within this context, before studying the N -point problem in the vector and matrix settings.

- **Is affine-invariance well defined on SPD matrices? A principled continuum of metrics**

Yann Thanwerdas and Xavier Pennec

Abstract: Symmetric Positive Definite (SPD) matrices have been widely used in medical data analysis and a number of different Riemannian metrics were proposed to compute with them. However, there are very few methodological principles guiding the choice of one particular metric for a given application. Invariance under the action of the affine transformations was suggested as a principle. Another

concept is based on symmetries. However, the affine-invariant metric and the recently proposed polar-affine metric are both invariant and symmetric. Comparing these two cousin metrics leads us to introduce much wider families: power-affine and deformed-affine metrics. Within this continuum, we investigate other principles to restrict the family size.

▪ Shape part transfer via semantic latent space factorization

Raphael Groskot, Laurent Cohen and Leonidas Guibas

Abstract: We present a latent space factorization that controls a generative neural network for shapes in a semantic way. Our method uses the segmentation data present in a shapes collection to explicitly factorize the encoder of a pointcloud autoencoder network, replacing it by several sub-encoders. This allows to learn a semantically-structured latent space in which we can uncover statistical modes corresponding to semantically similar shapes, as well as mixing parts from several objects to create hybrids and quickly exploring design ideas through varying shape combinations. Our work differs from existing methods in two ways: first, it proves the usefulness of neural networks to achieve shape combinations and second, adapts the whole geometry of the object to accommodate for its different parts.

15:10-15:40



Coffee break - Room: Amphi Bellonte Hall

15:40-17:20

Room: Amphi Bellonte

Session: **GEOMETRIC AND STRUCTURE PRESERVING DISCRETIZATIONS**

Chairs: **François Gay-Balmaz, Joël Bensoam**

▪ Variational discretization framework for geophysical flow models

Werner Bauer and François Gay-Balmaz

Abstract: We introduce a geometric variational discretization framework for geophysical flow models. The numerical scheme is obtained by discretizing, in a structure preserving way, the Lie group formulation of fluid dynamics on diffeomorphism groups and the associated variational principles. Being based on a discrete version of the Euler-Poincaré variational method, this discretization approach is widely applicable. We present an overview of structure-preserving variational discretizations of various equations of geophysical fluid dynamics, such as the Boussinesq, anelastic, pseudo-incompressible, and rotating shallow-water equations. We

verify the structure-preserving nature of the resulting variational integrators for test cases of geophysical relevance. Our framework applies to irregular mesh discretizations in 2D and 3D in planar and spherical geometry and produces schemes that preserve invariants of the equations such as mass and potential vorticity. Descending from variational principles, the discussed variational schemes exhibit a discrete version of Kelvin circulation theorem and show excellent long term energy behavior.

▪ Finite Element Methods for Geometric Evolution Equations

Evan Gawlik

Abstract: We study finite element methods for the solution of evolution equations in Riemannian geometry. Our focus is on Ricci flow and Ricci de Turck flow in two dimensions, where one of the main challenges from a numerical standpoint is to discretize the scalar curvature of a time-dependent Riemannian metric with finite elements. We propose a method for doing this which leverages Regge finite elements – piecewise polynomial symmetric (0,2)-tensors possessing continuous tangential-tangential components across element interfaces. In the lowest order setting, the finite element method we develop for two-dimensional Ricci flow is closely connected with a popular discretization of Ricci flow in which the scalar curvature is approximated with the so-called angle defect: 2π minus the sum of the angles between edges emanating from a common vertex. We present some results from our ongoing work on the analysis of the method, and we conclude with numerical examples.

▪ Local truncation error of low-order fractional variational integrators

Fernando Jimenez and Sina Ober-Blöbaum

Abstract: We study the local truncation error of the so-called fractional variational integrators, recently developed in [1,2] based on previous work by Riewe and Cresson [3,4]. These integrators are obtained through two main elements: the enlarging of the usual mechanical Lagrangian state space by the introduction of the fractional derivatives of the dynamical curves; and a discrete restricted variational principle, in the spirit of discrete mechanics and variational integrators [5]. The fractional variational integrators are designed for modelling fractional dissipative systems, which, in particular cases, reduce to mechanical systems with linear damping. All these elements are introduced in the paper. In addition, as original result, we prove (Section 3, Theorem 2) the order of local truncation error of the fractional variational integrators with respect to the dynamics of mechanical systems with linear damping.

▪ **A partitioned finite element method for the structure-preserving discretization of damped infinite-dimensional port-Hamiltonian systems with boundary control**

Anass Serhani, Denis Matignon and Ghislain Haine

Abstract: Many boundary controlled and observed Partial Differential Equations can be represented as port-Hamiltonian systems with dissipation, involving a Stokes-Dirac geometrical structure together with constitutive relations. The Partitioned Finite Element Method, introduced in Cardoso-Ribeiro et al. (2018), is a structure preserving numerical method which defines an underlying Dirac structure, and constitutive relations in weak form, leading to finite-dimensional port-Hamiltonian Differential Algebraic systems (pHDAE). Different types of dissipation are examined: internal damping, boundary damping and also diffusion models.

▪ **Geometry, Energy, and Entropy Compatible (GEEC) variational approaches to various numerical schemes for fluid dynamics**

Antoine Llor and Thibaud Vazquez-Gonzalez

Abstract: Since WW2, computer fluid dynamics has seen a staggering expansion of methods and applications fueled by the development of computer power. Now, of the main numerical approaches that have been explored over these years, only a few have become mainstream and make the vast majority of theoretical investigations in academia and practical usage in applications. These mostly hinge on concepts of finite volume discretization, monotonicity preservation, flux upwinding, and the analysis of the associated numerical dissipation processes - common tools here are the Riemann problem at cell interfaces and the Godunov scheme, more or less adapted from their original versions.

However, application to what looks as «niche» problems shows that these dominant approaches may not be as effective as generally accepted and have unduly benefited from a «winner-takes-it-all» effect. One of these problems is the simulation of isentropic flows which is actually «not-so-niche» as it is of high practical interest, especially in multi-fluid systems which involve complex energy transfers.

The present contribution aims at providing some perspective on CFD numerical schemes recently designed in order to better capture isentropic flows. The basic principle is that isentropic flow is geometric, i.e. potential (or internal) energy only depends on fluid density which in turn is defined by fluid element trajectories. A numerical scheme can thus be obtained by a variational, least action principle.

Corrections must be further added to enforce other properties such as energy conservation and positive dissipation. This Geometry, Energy, and Entropy Compatible approach (GEEC) is illustrated here on the historical von Neumann-Richtmyer Lagrangian scheme and on our recently developed multi-fluid Arbitrary Lagrangian-Eulerian scheme.

Room: G11

Session: OPTIMIZATION ON MANIFOLD

Chairs: Nicolas Brunel, Fabrice Gamboa

▪ **Canonical Moments for Optimal Uncertainty Quantification on a Variety**

Jérôme Stenger, Fabrice Gamboa, Merlin Keller and Bertrand Iooss

Abstract: The purpose of this work is to optimize an affine functional over positive measures. More precisely, we deal with a probability of failure (p.o.f). The optimization is realized over a set of distribution satisfying moment constraints, called moment set. The optimum is to be found on an extreme points of this moment set. Winkler's classification of those extreme points states they are finite discrete measures. The set of the support points of all discrete measures in the moment set is a manifold over which the p.o.f is optimized. We have discovered the structure of this manifold that is actually a variety (an algebraic manifold). It is the zero locus of polynomials defined thanks to the canonical moments. This reduces a highly constrained optimization over the moment set onto a constraint free manifold.

▪ **Computational investigations of an obstacle-type shape optimization problem in the space of smooth shapes**

Kathrin Welker and Daniel Luft

Abstract: We investigate and computationally solve a shape optimization problem constrained by a variational inequality of the first kind, a so-called obstacle-type problem, with a gradient descent and a BFGS algorithm in the space of smooth shapes. In order to circumvent the numerical problems related to the non-linearity of the shape derivative, we consider a regularization strategy leading to novel possibilities to numerically exploit structures, as well as possible treatment of the regularized variational inequality constrained shape optimization in the context of optimization on infinite dimensional Riemannian manifolds.

▪ Bézier curves and C^2 interpolation in Riemannian Symmetric Spaces

Chafik Samir and Ines Adouani

Abstract: We consider the problem of interpolating a finite set of observations at given time instant. In this paper, we introduce a new method to compute the optimal intermediate control points that define a C^2 interpolating Bézier curve. We prove this concept for interpolating data points belonging to a Riemannian symmetric spaces. The main property of the proposed method is that the control points minimize the mean square acceleration. Moreover, potential applications of fitting smooth paths on Riemannian manifold include applications in robotics, animations, graphics, and medical studies.

▪ A Formalization of The Natural Gradient Method for General Similarity Measures

Anton Mallasto, Tom Dela Haije and Aasa Feragen

Abstract: In optimization, the natural gradient method is well-known for likelihood maximization. The method uses the Kullback-Leibler divergence, corresponding infinitesimally to the Fisher-Rao metric, which is pulled back to the parameter space of a family of probability distributions. This way, gradients with respect to the parameters respect the Fisher-Rao geometry of the space of distributions, which might differ vastly from the standard Euclidean geometry of the parameter space, often leading to faster convergence. However, when minimizing an arbitrary similarity measure between distributions, it is generally unclear which metric to use. We provide a general framework that, given a similarity measure, derives a metric for the natural gradient. We then discuss connections between the natural gradient method and multiple other optimization techniques in the literature. Finally, we provide computations of the formal natural gradient to show overlap with well-known cases and to compute natural gradients in novel frameworks.

▪ The Frenet-Serret framework for aligning geometric curves

Nicolas Brunel and Juhyun Park

Abstract: Variations of the curves and trajectories in 1D can be analysed efficiently with functional data analysis tools. In particular, the main sources of variations in 1D curves have been identified as amplitude and phase variations. In particular, dealing with the latter gives rise to the problem of curve alignment and registration problems. It has been recognised that it is important to incorporate geometric features of the curves in developing statistical approaches to address such problems.

Extending these techniques to multidimensional curves is not

obvious, as the notion of multidimensional amplitude can be defined in multiple ways. We propose a framework to deal with the curve alignment in multidimensional curves as 3D objects. We propose a new distance between the curves that utilises the geometric information of the curves through the Frenet-Serret representation of the curves. This can be viewed as a generalisation of the elastic shape analysis based on the square root velocity framework. We develop an efficient computational algorithm to find an optimal alignment based on the proposed distance using dynamic programming.

Room: G13

Session: *GEOMETRY OF QUANTUM STATES*

Chairs: *Florio Maria Ciaglia, Jan Naudts*

▪ When geometry meets psycho-physics and quantum mechanics : Modern perspectives on the space of perceived colors

Michel Berthier and Edoardo Provenzi

Abstract: We discuss some modern perspectives about the mathematical formalization of colorimetry, motivated by the analysis of a groundbreaking, yet poorly known, model of the color space proposed by H.L. Resnikoff and based on differential geometry. In particular, we will underline two facts: the first is the need of novel, carefully implemented, psycho-physical experiments and the second is the role that Jordan algebras may have in the development of a more rigorously founded colorimetry.

▪ Quantum statistical manifolds: The finite-dimensional case

Jan Naudts

Abstract: Quantum information geometry studies families of quantum states by means of differential geometry. A new approach is followed. The emphasis is shifted from a manifold of strictly positive density matrices to a manifold M of faithful quantum states on a von Neumann algebra of bounded linear operators working on a Hilbert space. In order to avoid technicalities the theory is developed for the algebra of n -by- n matrices. A chart is introduced which is centered at a given faithful state. It maps the manifold M onto a real Banach space of self-adjoint operators belonging to the commutant algebra. The operator labeling any state of M also determines a tangent vector at the center of M along the exponential geodesic in the direction of that state. A link with the theory of the modular automorphism group is worked out. Explicit expressions for the chart can be derived in terms of the modular conjugation and the relative modular operators.

▪ Generalized Gibbs Ensembles in Discrete Quantum Gravity

Goffredo Chirco and Isha Kotecha

Abstract: Maximum entropy principle and Souriau's symplectic generalization of Gibbs states have provided crucial insights leading to extensions of standard equilibrium statistical mechanics and thermodynamics. In this brief contribution, we show how such extensions are instrumental in the setting of discrete quantum gravity, to provide a covariant statistical framework for the emergence of continuum spacetime from discrete quantum gravity. We discuss the significant role played by information-theoretic characterizations of equilibrium, surrounding entropy maximization and a further generalization of Souriau's Gibbs states. As an illustrative example, we consider the symplectic Gibbs state description of the geometry of a tetrahedron and its quantization, thereby providing a statistical description of the characterizing quantum of space in quantum gravity.

▪ On the notion of composite system, classical and quantum

Florio Maria Ciaglia, Giuseppe Marmo and Alberto Ibort

Abstract: We will analyze the notion of composite systems in the framework of p -convex spaces of which the spaces of quantum states and classical probability distributions are specific instances. We will see that, once suitable compatibility conditions are imposed in order to implement the notion of composite system in this theoretical framework, the notion of algebraic tensor product of the underlying vector spaces presents itself as a «minimal representation» of a composite systems.

17:20-18:00

Room: Amphi Bellonte Hall

Poster session

Chair: [Pierre Baudot](#)

▪ Parcels of Universe and Stochastic Processes

Marco Frasca and Alfonso Farina

Abstract: We consider Brownian motion on quantized non-commutative Riemannian manifolds and show how a set of stochastic processes on sets of complex numbers can be devised. This class of stochastic processes are show to yield at the outset a Chapman-Kolmogorov equation with a complex diffusion coefficient that can be straightforwardly reduced to the Schrödinger equation. The existence of these processes has been recently shown numerically.

In this work we provide an analogous support for the existence of the Chapman-Kolmogorov-Schrödinger equation for them. Besides, it is numerically seen as a Wick rotation can change the heat kernel into the Schrödinger one.

▪ Geometry of tangent and cotangent bundles on statistical manifolds

Tat Dat To and Stephane Puechmorel

Abstract: This note surveys some results on the geometric structure on the tangent bundle and cotangent bundle of statistical manifolds. We also study the completeness of the tangent bundle with respect to the Sasaki metrics induced from the statistical structures.

▪ Bayesian statistical models and Bayesian machine learning

Hong Van Le

Abstract: We revisit the notion of probabilistic mappings. Then we introduce the notion of a Bayesian statistical model that formalizes the notion of a parameter space with a given prior distribution in Bayesian machine learning. We give a general Bayes' formula and revisit Dirichlet measures using Bayesian statistical models and probabilistic mappings.

▪ Evolution of Quantum States, a Lagrangian description

Fabio Di Cosmo, Giuseppe Marmo and Luca Schiavone

Abstract: The space of quantum states is a stratified manifold. It carries two main stratifications, one provided by the rank of the state, the other one is given by its spectrum. In the former case the stratum is an orbit of the complex special linear group, while in the latter it is an orbit of the unitary group. The action of the special linear group is not linear but carries many interesting properties with respect to convexity, moreover it is the maximal group contained in the semigroup of completely positive trace preserving maps. In this contribution we would like to address the possibility that the one-parameter subgroups of either one of the two groups may be given a Lagrangian description. Apart from the possibility of importing into this quantum description the traditional tools of Lagrangian mechanics, such as the link between Lagrangian symmetries and constants of the motion or the possibility of performing a quantization (which would be a second quantization in this case), we investigate such an approach for mainly two reasons. From one point of view, these one-parameter subgroups could be associated with the Lagrangian formulation of one-parameter subgroups on suitable covering-spaces, for instance when obstructions occur. We call this procedure unfolding and reduction: a new dynamics

is defined on an enlarged carrier space and, afterwards, the original description is recovered by means of a suitable projection. For instance, a Lagrangian function on the space of pure states of a N -level quantum system, which correspond to the complex projective Hilbert space $CP(N-1)$, cannot be globally defined for one-parameter subgroups of the unitary group. However, unfolding to the whole Hilbert space allows to overcome this obstruction and a following reduction to the projective space reproduces the desired one-parameter group of transformations.

From another point of view, functions can be restricted to submanifolds, and, consequently, starting with a one-parameter group of transformations on a manifold, which can be given a Lagrangian description, it becomes possible to define a "restricted dynamical system" on embedded submanifolds. Within the context of quantum systems, this procedure would amount to restrict a dynamics, originally defined on generic mixed states, to a subclass of "trial" states which can be of interest for some subsequent analysis.

▪ Weighted Fshape Model in Computational Anatomy and Hyperparameter Estimation

Tomoki Tanaka, Tomonari Sei, Shouhei Hanaoka, Hidekata Hontani and Akinobu Shimizu

Abstract: Computational anatomy is the field of mathematical structures describing human organs. In computational anatomy, the fshape model was recently proposed, by which we can consider both shape and color of an organ. However, importance of positions within the organ cannot be put into the fshape model. In this paper, we propose a weighted fshape model that weights an evaluation function by position.

In addition, we should choose regularization parameters in the fshape model. Therefore, we also propose an estimation method of the parameters by introducing a statistical model of the fshape model and using a hyperparameter estimation method, for example, an EM algorithm.

▪ Alternative divergence functions from alternative Hermitian structures on finite dimensional complex projective spaces

Alessandro Zampini, Florio Maria Ciaglia and Giuseppe Marmo

Abstract: Consider a smooth manifold M on which a linear structure (formulated via a Euler vector field Δ on M) and a complex linear structure (i.e. a $(1,1)$ tensor J on M with $J^2 = -id$) are defined. This allows the identifications $M \cong \mathbb{R}^{2n}$. The vector field Δ commutes with J , so the quotient of \mathbb{R}^{2n} under the action of the involutive

distribution generated by $\{\Delta, J\}$ provides a principal fibration on a basis which is the complex projective space. It is then clear that CP^{n-1} depends only on the linear complex structure on \mathbb{C}^n : the embedding $CP^{n-1} \hookrightarrow \mathbb{C}^n$ (with $u \in \mathbb{C}^n$ the vector space of n -dimensional Hermitian matrices, i.e. the dual to the Lie algebra $\mathfrak{u}(n)$ of the unitary group $U(n)$) depends indeed on the Hermitian structure h on \mathbb{C}^n (with $h(u,v) = \langle u, v \rangle$ for $u, v \in \mathbb{C}^n$) which induces (see [CIMM15, MZ2018] for a review) an Hermitian structure on CP^{n-1} . Since the action of $SL(n, \mathbb{C})$ leaves both Δ and J invariant, it induces an action on CP^{n-1} , which reads, for $T \in SL(n, \mathbb{C})$, $h_T(u,v) = \langle u, v \rangle_T = \langle u, T^{-1}Tv \rangle$; when $T \in SU(n)$, then $h_T = h$. When $T \in SL(n, \mathbb{C})/SU(n)$, the Hermitian structure h_T has changed.

A linear vector field on M is said to describe a quantum dynamics (see [gfd] for a review) if a Hermitian structure h on M exists, such that the flow generated by X is unitary with respect to h . In this case, the vector field X is generated by a Hamiltonian operator H . Notice that this amounts to say that X is both a Killing vector field and a Hamiltonian vector field, with Hamiltonian function f_H on CP^{n-1} , with respect to the compatible Euclidean g and the symplectic structures given by the real and the imaginary part of $h = g + i\omega$, with $g(u,v) = \langle u, v \rangle$; $g(Ju, Jv) = g(u,v)$; $(Ju, Jv) = \langle u, v \rangle$. The probabilistic interpretation of quantum mechanics leads to consider that the set of pure states is described by the complex projective space. In [CIMM15] it is described how a reduction procedure along such a principal fibration and an unfolding procedure associated to a suitable momentum map allow to describe the Kähler geometry of $S = \mathbb{C}N/O/\mathbb{C}O$

If the quantum dynamics satisfies the compatibility condition $[H, T] = 0$ with $T \in SL(n, \mathbb{C})/SU(n)$, then the dynamics (the flow) turns out to be unitary with respect to the Hermitian structure h_T . With respect to the compatible Euclidean and symplectic tensors $h_T = g_T + i\omega_T$, the 2-dynamics is Hamiltonian with Hamiltonian function $f(T)H$ on the same projective space CP^{n-1} . Such quantum dynamics is then unitary with respect to different Hermitian structures. The interest of this analysis goes both in the direction of analysing the complete integrability of such dynamics, and towards applications to geometric information theory. It is indeed interesting to notice that the action functional $HS = \int dt (p \dot{q} - f(T))$ depends on the Hamiltonian function, so alternative Hermitian descriptions provide different divergence functions [AN2000].

References

- [AN2000] S. Amari, H. Nagaoka, Methods of information geometry, Oxford Univ. Press 2000;
- [CIMM15] J.F. Cariñena, A. Ibort, G. Marmo, G. Morandi, Geometry from Dynamics, classical and quantum, Springer 2015;
- [MZ2018] G. Marmo, A. Zampini, Kähler geometry on projective spaces via reduction and unfolding, Rend. Mat. Appl. 39 (2018) 329-345;

▪ **Derivation of the Fisher Information Matrix for the Complex Gaussian Distribution**, Deepika Kumari and Luigi Malagò

Abstract: The complex Gaussian distribution is a statistical model for complex random variables whose real and imaginary parts are normal distributed, which finds multiple applications in many fields of applied analysis, physics, and engineering. Complex Gaussian distributions are commonly parameterized by a complex mean vector, a Hermitian positive-definite covariance matrix and a symmetric complex relation matrix. In this paper we focus on the sub-family of complex Gaussian distributions characterized by zero relation matrix, which includes circularly symmetric complex random variables as special case, when the mean is set to zero. In the first part of the paper we provide explicit formulae for the entries of the Fisher matrix in three different parameterizations: the standard mean and covariance, and the natural and the expectation parameters of the exponential family, which are dual parameterizations of interest in information geometry. In the second part of this work we introduce conjugate parameters for complex Gaussians, and by using Wirtinger derivatives, we define a corresponding form for the Fisher information matrix based on conjugate scores. Finally we show that the well known formula by Miller for the Fisher information matrix for Gaussian distributions can be generalized to conjugate parameterizations. Compared to the real case, in the conjugate parameters the Fisher information matrix appears to have a more direct formulation.

▪ **Information Geometry and Sparse Goodness-of-Fit Testing**
Frank Critchley, Paul Marriott and Radka Sabolova

Abstract: This paper is part of an exploration of the statistical behaviour of divergence based goodness-of-fit testing outside the usual asymptotic regimes in the case of sparsity. We work with the Information Geometry of extended exponential families since the data, typically, lies on the boundary of its sample space. We explore the idea of defining an extended divergence for which p-values can be computed and start the exploration of a natural information decomposition across a wide set of divergences in the sparse case.

▪ **Geometric Hydrodynamics of Compressible Fluids**
Boris Khesin, Gerard Misiolek and Klas Modin

Abstract: We shall discuss a framework for geometric compressible hydrodynamics. Please notice that we do not wish to submit a full paper for the proceedings, just an abstract for our talk.

▪ **Applications in Robotics with probability on Riemannian Manifolds**
Sylvain Calinon and Noemie Jaquier

Abstract: This paper presents an overview of applications in robotics exploiting statistics and Riemannian geometry. We first list the most common manifolds used in robotics. Several techniques are then described based on weighted averaging and Gaussian-like distributions on manifolds, including clustering, regression, information fusion, planning and control problems. Further perspectives are finally discussed with suggestions of promising research directions.

▪ **A Canonical Invariant Of Statistical Models**
Michel Nguiffo Boyom and Frederic Barbaresco

Abstract: We are concerned with Fisher information gp of a functional statistical models $[E, \pi, M, p]$. The kernel of g, K_p is an Lie algebroid. By involving a Koszul connection in K_g , we introduce a canonical 2-cohomology class of order ≤ 2 , $[L_p]$ whose vanishing is equivalent to the regularity of g_p . We also remind a numerical invariant $rb(g)$ which measure how far from being Hessian is a metric tensor g .

▪ **Robust Multiscale Decomposition of the Intrinsic Dimension of 3D Curves**
Minh Son Phan, Katherine Matho, Lamiae Abdeladim, Jean Livet, Emmanuel Beaufreire and Anatole Chessel

Abstract: 3D curves appear in many research fields, such as object motion in robotics, single particle tracking or neuron axonal paths in biology. While embedded in 3D, they often present portions which can appropriately be described as linear or planar. Being able to identify these portions and their spatial dimensionality is key to understanding the processes that create and shape them. In this work we propose algorithms based on curvature and torsion scale spaces to compute such intrinsic dimension decomposition. We evaluate them on simulated traces and apply them to analyse intrinsic information from axonal trajectories traced in the mouse brain.

▪ **Intrinsic Shrinkage Estimator on Matrix Lie Groups with Applications**
Chun-Hao Yang and Baba Vemuri

Abstract: Data or measurements in non-Euclidean spaces are commonly encountered in many fields of Science and Engineering.

For instance, in Robotics, attitude sensors capture orientation which is an element of a Lie group. In the recent past, several researchers have developed methods that take into account the geometry of Lie Groups in designing filtering algorithms for tracking in nonlinear spaces. Many of these algorithms estimate parameters of a state-space model used to characterize the filters. Maximum likelihood estimators (MLE) are quite commonly used for such tasks and it is well known in the field of statistics that shrinkage estimators dominate the MLE in a mean-squared sense. In this paper we present a novel shrinkage estimator for data residing in Lie groups, specifically, matrix Lie groups. We empirically demonstrate the dominance of the shrinkage estimates over MLE via several synthetic data experiments as well as in multi-object tracking/localization.

▪ **Chen invariants in Legendrian submanifolds of holomorphic statistical manifolds**

Michel Nguiffo Boyom, Pooja Bansal, Azeb Alghanemi and Mohammad Hasan Shahid

Abstract: In this work with the glimpse of Optimization method, we construct Chen-Ricci inequality for Legendrian submanifolds of holomorphic statistical manifolds. A necessary and sufficient condition for the equality case is also described.

▪ **The Poincaré-Boltzmann Machine: passing the information between disciplines**

Pierre Baudot

Abstract: This note presents the computational methods of information cohomology applied to genetic expression in [93,11,12] and proposes its interpretations in terms of statistical physic and machine learning. In order to further underline the Hochschild cohomological nature of information functions and chain rules, following [9,10,97], the cohomology in low degrees is computed to show more directly that the k multivariate mutual-informations (I_k) are k -coboundaries. The k -cocycles condition corresponds to $I_k = 0$, which generalize statistical independence to arbitrary dimension k [11]. Hence the cohomology can be interpreted as quantifying the statistical dependences and the obstruction to factorization. The topological approach allows to investigate Shannon's information in the multivariate case without the assumptions of independent identically distributed variables. We develop the computationally tractable subcase of simplicial information cohomology represented by entropy H_k and information I_k landscapes and their respective paths. The I_1 component defines a self-internal energy functional U_k , and $(-1)^k I_k, k \geq 2$ components define the contribution to a free

energy functional G_k (the total correlation) of the k -body interactions. The set of information paths in simplicial structures is in bijection with the symmetric group and random processes, provides a trivial topological expression of the 2nd law of thermodynamic. The local minima of free-energy, related to conditional information negativity, and conditional independence, characterize a minimum free energy complex. This complex formalizes the minimum free-energy principle in topology, provides a definition of a complex system, and characterizes a multiplicity of local minima that quantifies the diversity observed in biology. I give an interpretation of this complex in terms of frustration in glass and of Van Der Waals k -body interactions for data points (without symplectic structure).

18:00-19:00

Room: **Amphi Bellonte**

Keynote Speaker: **Elena Celledoni**

Structure preserving algorithms for geometric numerical integration

20:00-22:00



Gala dinner at Hôtel Dieu

// Thursday, August 29th

09:00-09:30

Registration desk – Badge withdrawing

09:30-10:00

Room: Amphi Bellonte

Gilles Baroin

From Circle to Hyper-spheres when Music goes 4-D

Short biography

Gilles Baroin started learning music with his father at school at the age of 5. He has a master degree in mechanics and computer graphics, a Ph-D in musicology with summa cum laude. Specialized in geometric visualization of musical structures and processes, designer of Planet-4D model and hyperspheres, he regularly collaborates with artists, musicians, tuners and composers to develop new models in 2D or 4D spaces. Producer of the first animated movie of an atonal piece from Webern bagatelle in 4D at IRCAM Paris, he participates regularly to mathematical events, seminars and artistic performances, offers talks in universities and conservatories worldwide. Member of the Society for Mathematic and Computation in Music and European Society for Mathematics and Arts, associated researcher in mathematics at the French National School for Civil Aviation, and in musicology at LLa-Créatis Laboratory in the federal University of Toulouse. www.MatheMusic.net

Some venues

Conservatory of Shanghai, Centre Pompidou Paris, Royal Conservatory of Madrid, Mc Gill University Montreal, Queen Mary University of London, Maison de Fermat Beaumont, University of Ljubljana, Centre Bellegarde Toulouse, University of Athens, Città de Firenze, University of Strasbourg, Mains Museum Heidelberg, ...

10:00-12:00

Room: Amphi Bellonte

Session: **PROBABILITY ON RIEMANNIAN MANIFOLDS**Chairs: **Marc Arnaudon, Ana Bela Cruzeiro**
▪ The Riemannian barycentre as a proxy for global optimisation
 Salem Said and Jonathan Manton

Abstract: Let M be a simply-connected compact Riemannian symmetric space, and U a twice-differentiable function on M , with unique global minimum at x^* , where $d(\cdot, \cdot)$ denotes Riemannian distance. The following original result is proved : if U is invariant by geodesic symmetry about M . The idea of the present work is to replace the problem of searching for the global minimum of U , by the problem of finding the Riemannian barycentre of the Gibbs distribution $PT \exp(-U/T)$. In other words, instead of minimising the function U itself, to minimise $ET(x) = \int d^2(x,z)PT(dz)x^*$, then for each $< 12rcx$ (rcx the convexity radius of M), there exists T such that $T \leq T$ implies ET is strongly convex on the geodesic ball $]B(x^*, ,$ and x^* is the unique global minimum of ET . Moreover, this T can be computed explicitly. This result gives rise to a general algorithm for black-box optimisation, which is briefly described, and will be further explored in future work.

▪ Hamiltonian Monte Carlo On Lie Groups and Constrained Mechanics on Homogeneous Manifolds

Alessandro Barp

Abstract: In this paper we derive the correspondence between the various formulations of Hamiltonian mechanics that can be defined over a Lie group, and their induced Hamiltonian Monte Carlo algorithms. Using the Euler-Arnold formulation, we extend the HMC scheme to non-compact Lie groups, by choosing metrics with appropriate invariances.

Finally we explain how mechanics on homogeneous spaces can be formulated as a constrained system on Lie groups, and how in some cases the constraints can be naturally handled by the symmetries of the Hamiltonian.

▪ On the Fisher-Rao information metric in the space of normal distributions

Julianna Pinele, Sueli Costa and João Strapasson

Abstract: The Fisher-Rao distance between two probability distribution functions, as well as other divergence measures, is related to entropy and is in the core of the research area of information geometry. It can provide a framework and enlarge the perspective of analysis for a wide variety of domains, such as statistical inference, image processing (texture classification and inpainting), clustering processes and morphological classification. We present here a compact summary of results regarding the

Fisher-Rao distance in the space of multivariate normal distributions including some historical background, closed forms in special cases, bounds, numerical approaches and references to recent applications.

▪ **Simulation of Conditioned Diffusions on the Flat Torus**

Mathias Højgaard Jensen, Anton Mallasto and Stefan Sommer

Abstract: Diffusion processes are fundamental in modelling stochastic dynamics in natural sciences. Recently, simulating such processes on complicated geometries has found applications for example in biology, where toroidal data arises naturally when studying the backbone of protein sequences, creating a demand for efficient sampling methods. In this paper, we propose a method for simulating diffusions on the flat torus, conditioned on hitting a terminal point after a fixed time, by considering a diffusion process in \mathbb{R}^2 which we project onto the torus. We contribute a convergence result for this diffusion process, translating into convergence of the projected process to the terminal point on the torus. We also show that under a suitable change of measure the Euclidean diffusion is locally a Brownian motion.

▪ **Towards parametric bi-invariant density estimation on $SE(2)$**

Emmanuel Chevallier

Abstract: This paper aims at describing a novel framework for bi-invariant density estimation on the group of planar rigid motion $SE(2)$. Probability distributions on the group are constructed from distributions on tangent spaces pushed to the group by the exponential map. The exponential mapping on Lie groups presents two key particularities: it is compatible with left and right multiplications and its Jacobian can be computed explicitly. These two properties enable to define probability densities with tractable expressions and bi-invariant procedures to estimate them from a set of samples. Sampling from these distributions is easy since it is sufficient to draw samples in Euclidean tangent spaces. This paper is a preliminary work and the convergences of these estimators are not studied.

Room: G11

Session: WASSERSTEIN INFORMATION GEOMETRY / OPTIMAL TRANSPORT

Chairs: Guido Montufar, Wuchen Li

▪ **Affine Natural Proximal Learning**

Wuchen Li, Alex Lin and Guido Montufar

Abstract: We revisit the natural gradient method for learning in statistical manifolds.

We consider the proximal formulation and obtain a closed-form approximation of the proximity term over an affine subspace of functions in the Legendre dual formulation. We consider two important types of statistical metrics, namely the Wasserstein and Fisher-Rao metrics, and introduce numerical methods for high dimensional parameter spaces.

▪ **Parametric Fokker-Planck equation**

Wuchen Li, Shu Liu, Hongyuan Zha and Haomin Zhou

Abstract: We derive the Fokker-Planck equation on the parametric space. It is the Wasserstein gradient flow of relative entropy on the statistical manifold. We pull back the PDE to a finite dimensional ODE on parameter space. An analytical example and numerical examples in one-dimensional sample space are presented.

▪ **Multi-marginal Schroedinger bridges**

Yongxin Chen, Giovanni Conforti, Tryphon Georgiou and Luigia Ripani

Abstract: We consider the problem to identify the most likely flow in phase space, of (inertial) particles under stochastic forcing, that is in agreement with spatial (marginal) distributions that are specified at a set of points in time. The question raised generalizes the classical Schrödinger Bridge Problem (SBP) which seeks to interpolate two specified end-point marginal distributions of overdamped particles driven by stochastic excitation. While we restrict our analysis to second-order dynamics for the particles, the data represents partial (i.e., only positional) information on the flow at multiple time-points. The solution sought, as in SBP, represents a probability law on the space of paths this closest to a uniform prior while consistent with the given marginals. We approach this problem as an optimal control problem to minimize an action integral a la Benamou-Brenier, and derive a time-symmetric formulation that includes a Fisher information term on the velocity field. We underscore the relation of our problem to recent measure-valued splines in Wasserstein space, which is akin to that between SBP and Optimal Mass Transport (OMT). The connection between the two provides a Sinkhorn-like approach to computing measure-valued splines. We envision that interpolation between measures as sought herein will have a wide range of applications in signal/images processing as well as in data science in cases where data have a temporal dimension.

▪ Hopf-Cole transformation and Schrödinger problems

Flavien Leger and Wuchen Li

Abstract: We study generalized Hopf-Cole transformations motivated by the Schrödinger bridge problem. We present two examples of canonical transformations, including a Schrödinger problem associated with a quadratic Rényi entropy.

▪ Curvature of the manifold of fixed-rank positive-semidefinite matrices endowed with the Bures-Wasserstein metric

Estelle Massart, Julien M. Hendrickx and P.-A. Absi

Abstract: We consider the manifold of fixed-rank positive-semidefinite matrices, seen as a quotient of the set of full-rank rectangular matrices by the orthogonal group. The resulting distance coincides with the Wasserstein distance between centered degenerate Gaussian distributions. We obtain expressions for the Riemannian curvature tensor and the sectional curvature of the manifold. We also provide tangent vectors spanning planes associated with the extreme values of the sectional curvature.

Room: G13

Session: GEOMETRIC SCIENCE OF INFORMATION LIBRARIES

Chairs: *Nina Miolane, Alice Le Brigant*

▪ Second-order networks in PyTorch

Daniel Brooks, Olivier Scwander, Frédéric Barbaresco, Jean-Yves Schneider and Matthieu Cord

Abstract: Classification of Symmetric Positive Definite (SPD) matrices is gaining momentum in a variety machine learning application fields. In this work we propose a Python library which implements neural networks on SPD matrices, based on the popular deep learning framework Pytorch.

▪ Symmetric Algorithmic Components for Shape Analysis with Diffeomorphisms

Nicolas Guigui, Shuman Jia, Maxime Sermesant and Xavier Pennec

Abstract: In computational anatomy, the statistical analysis of temporal deformations and inter-subject variability relies on shape registration. However, the numerical integration and optimization required in diffeomorphic registration often lead to important numerical errors. In many cases, it is well known that the error can be drastically reduced in the presence of a symmetry. In this work, the leading idea is to approximate the space of deformations and images

with a possibly non-metric symmetric space structure using an involution, with the aim to perform parallel transport. Through basic properties of symmetries, we investigate how the implementations of a midpoint and the involution compare with the ones of the Riemannian exponential and logarithm on diffeomorphisms and propose a modification of these maps using registration errors. This leads us to identify transvections, the composition of two symmetries, as a mean to measure how far from symmetric the underlying structure is. We test our method on a set of 138 cardiac shapes and demonstrate improved numerical consistency in the Pole Ladder scheme.

▪ Softwares for Computational Optimal Transport

Gabriel Peyré

Abstract: In this talk, I will present some state of the art libraries for computational optimal transport, with an emphasis on too approaches which have recently gained a lot of interest: semi-discrete optimal transport and entropic regularization (aka Sinkhorn). I will cover in particular:

- The numerical tours (<http://www.numerical-tours.com/python/>) which can be used for education purpose, and showcase detailed simple pedagogical implementations of these two lines of works.
- POT: Python Optimal Transport (<https://pot.readthedocs.io/>) which contains basic implementation of standard OT solvers (including simplex and Sinkhorn).
- KeOps (<https://www.kernel-operations.io>) which is a state of the art implementation of entropic regularization method, which streams extremely well on GPU architectures. More material and resources are available from <https://team.inria.fr/ecoleceainriaedf/>
- pysdot (<https://github.com/sd-ot/pysdot>) and Geogram (<http://alice.loria.fr/software/geogram/doc/html/index.html>) which are Python and C++ library which integrate state of the art computational geometry procedure to compute semi-discrete OT.

▪ Geomstats library

Nina Miolane, Alice Le Brigant

Abstract: We present geomstats, an open source python package that performs computations on manifolds such as hyperspheres, hyperbolic spaces, spaces of symmetric positive definite matrices, Lie groups of transformations and Stiefel manifolds. The implementation is object-oriented and extensively unit-tested. The manifolds come with families of Riemannian metrics, each associated with its Exponential/Logarithm maps, geodesics and geodesic distances. The operations are vectorized for batch computations and available with

both numpy, tensorflow and pytorch backends, which allows GPU computing. The presentation of the package will be illustrated with some applications in air traffic control and fetal brain reconstruction from MRIs.

12:00-13:00

 Lunch break

13:00-14:00

Room: Amphi Bellonte

Keynote Speaker: *Gabriel Peyré*

Optimal Transport for Machine Learning

14:00-14:30

Room: Amphi Bellonte

Closing Session: *Frédéric Barbaresco, Frank Nielsen, Patrick Sénac*

// Committees

Steering committee

- **Valérie Alidor**

SEE

- **Frédéric Barbaresco**

THALES LAS FRANCE SAS <http://www.thalesgroup.com>

- **Ludovic D'Estampes**

ENAC

- **Marianne Emorine**

SEE

- **Thierry Klein**

ENAC

- **Alice Le Brigant**

Université de Bordeaux

- **Florence Nicol**

Ecole Nationale de l'Aviation Civile

- **Frank Nielsen**

Ecole Polytechnique, France <http://www.lix.polytechnique.fr/~nielsen/>

- **Stephane Puechmorel**

ENAC

- **Tat-Dat To**

ENAC

Scientific committee

- **Pierre-Antoine Absil**

University of Louvain, Belgium <http://sites.uclouvain.be/absil/>

- **Bijan Afsari**

Johns Hopkins University, USA <http://www.cis.jhu.edu/~bijan/>

- **Stéphanie Allasonnière**

Ecole Polytechnique

- **Jesus Angulo**

Mines ParisTech, France <http://cmm.ensmp.fr/~angulo/>

- **Jean-Pierre Antoine**

Université catholique de Louvain

- **John Armstrong**

King's College London

- **Marc Arnaudon**

Université de Bordeaux, France <http://www.math.u-bordeaux1.fr/~marnaudon/>

- **Dena Asta**

Carnegie Mellon University, USA

- **Anne Auger**

Inria

- **Nihat Ay**

Max Planck Institute for Mathematics in the Sciences

- **Roger Balian**

Academy of Sciences, France https://en.wikipedia.org/wiki/Roger_Balian

- **Frédéric Barbaresco**
THALES LAS FRANCE SAS <http://www.thalesgroup.com>
- **Pierre Baudot**
Max Planck Institute for Mathematic in the Sciences <http://www.mis.mpg.de/jjost/members/pierre-baudot.html>
- **Martin Bauer**
Florida State University <https://www.math.fsu.edu/~bauer/>
- **Daniel Bennequin**
Paris-Diderot University <http://webusers.imj-prg.fr/~daniel.bennequin/>
- **Joël Bensoam**
ircam
- **Yannick Berthoumieu**
IMS Université de Bordeaux, France <https://sites.google.com/site/berthoumieuims/>
- **Jérémie Bigot**
Université de Bordeaux
- **Silvère Bonnabel**
Mines-Paristech <http://www.silvere-bonnabel.com/>
- **Michel Boyom**
Université de Montpellier, France <http://www.i3m.univ-montp2.fr/>
- **Michel Broniatowski**
University of Pierre and Marie Curie, France <http://www.lsta.upmc.fr/Broniatowski/>
- **Marius Buliga**
Simion Stoilow Institute of Mathematics of the Romanian Academy
- **Alain Chenciner**
IMCCE & Université Paris 7
- **Laurent Cohen**
Université Paris Dauphine
- **Patricia Conde céspeles**
Université Pierre et Marie Curie
- **Ana Cruzeiro**
IST, Université de Lisbonne
- **Géry De Saxcé**
Université des Sciences et des Technologies de Lille, France <http://www.univ-lille1.fr/>
- **Remco Duits**
Eindhoven university of technology
- **Stanley Durrleman**
INRIA, France <https://who.rocq.inria.fr/Stanley.Durrleman/index.html>
- **Diala Ezzeddine**
Laboratoire ERIC
- **Alfred Galichon**
New York University, USA
- **Fabrice Gamboa**
Institut Mathématique de Toulouse
- **François Gay-Balmaz**
CNRS, ENS-ULM
- **Jean-Pierre Gazeau**
Université Paris-Diderot
- **Mark Girolami**
Imperial College London
- **Douglas Gonçalves**
Universidade Federal de Santa Catarina <http://mtm.ufsc.br/~douglas>
- **Matsuzoe Hiroshi**
Nagoya Institute of Technology
- **Susan Holmes**
Stanford University
- **Jérémie Jakubowicz**
Institut Mines Telecom, France <http://www-publicit-sudparis.eu/~jakubowi/>
- **Nicolas Le Bihan**
Université de Grenoble, CNRS, France - University of Melbourne, Australia <http://www.gipsa-lab.grenoble-inp.fr/~nicolas.le-bihan/>
- **Jean Lerbet**
Université Paris Saclay
- **Luigi Malagò**
Romanian Institute of Science and Technology, Romania <http://www.luigimalago.it/>
- **Jonathan Manton**
The University of Melbourne <http://people.eng.unimelb.edu.au/jmanton/>
- **Gaetan Marceau Caron**
Thales Theresis
- **Matilde Marcolli**
Caltech
- **Jean-François Marcotorchino**
Thales, France <https://www.thalesgroup.com/>
- **Charles-Michel Marle**
Université Pierre et Marie Curie, France <http://charles-michel.marle.pagesperso-orange.fr/>
- **Juliette Mattioli**
THALES RESEARCH AND TECHNOLOGY <https://www.thalesgroup.com/en>
- **Bertrand Maury**
Université Paris Sud, France <http://www.math.u-psud.fr/~maury/>
- **Quentin Mérigot**
Université Paris-Dauphine / CNRS
- **Nina Miolane**
Inria
- **Jean-Marie Mirebeau**
Univ Paris-Sud, CNRS, Univ Paris-Saclay

- **Ali Mohammad-Djafari**

Supelec, CNRS, France <http://djafari.free.fr/>

- **Antonio Mucherino**

IRISA, Université de Rennes 1

- **Florence Nicol**

Ecole Nationale de l'Aviation Civile

- **Frank Nielsen**

Ecole Polytechnique, France <http://www.lix.polytechnique.fr/~nielsen/>

- **Richard Nock**

*Université des Antilles et de la Guyane, France - NICTA, Australia
<http://www.univ-ag.fr/rnock/index.html>*

- **Yann Ollivier**

Université Paris Sud, France <http://www.yann-ollivier.org/>

- **Steve Oudot**

INRIA

- **Maks Ovsjanikov**

Ecole Polytechnique

- **Pierre Pansu**

Université Paris-Sud

- **Xavier Pennec**

INRIA, France <http://www-sop.inria.fr/members/Xavier.Pennec/>

- **Michel Petitjean**

Université Paris Diderot, CNRS, France <http://petitjeanmichel.free.fr/itoweb.petitjean.html>

- **Marion Pilté**

Mines ParisTech

- **Giovanni Pistone**

Collegio Carlo Alberto, Moncalieri, Italie <http://www.giannidiordestino.it/>

- **Radmila Pribic**

Thales Nederland

- **Stephane Puechmorel**

ENAC

- **Olivier Rioul**

Telecom ParisTech, France <http://perso.telecom-paristech.fr/~rioul/>

- **Salem Said**

Laboratoire IMS

- **Carola-bibiane Schönlieb**

University of Cambridge

- **Olivier Schwander**

UPMC, Paris, France <http://www-connex.lip6.fr/~schwander>

- **Rodolphe Sepulchre**

Cambridge University, Department of Engineering, UK

- **Fátima Silva Leite**

University of Coimbra, Portugal <https://www.mat.uc.pt/~fleite/>

- **Stefan Sommer**

University of Copenhagen

- **Dominique Spehner**

Université Grenoble Alpes <https://www-fourier.ujf-grenoble.fr/~spehner/>

- **Wolfgang Stummer**

Department of Mathematics, University of Erlangen-Nurnberg

- **Barbara Trivellato**

Politecnico di Torino, Italy <http://calvino.polito.it/~trivellato/>

- **Alain Trouvé**

ENS Cachan, France <http://atrouve.perso.math.cnrs.fr/>

- **Barbara Tumpach**

Laboratoire Painlevé <http://math.univ-lille1.fr/~tumpach/Site/home.html>

- **Geert Verdoolaeghe**

Ghent University

- **René Vidal**

University of Michigan

- **Jun Zhang**

Ann Arbor, USA

Local organizers

- **Gilles Baroin**

ENACLab, Research and Communication

- **Murat Bronz**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Ludovic d'Estampes**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Thierry Klein**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Alice Le Brigant**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Simon Le Sénéchal**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Georges Mykoniatis**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Florence Nicol**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Stephane Puechmorel**

Scientific advisor, ENAC, Toulouse France

- **Tat Dat Tô**

ENACLab, DEVI research team, ENAC, Toulouse France

- **Catherine Nerestant**

Graphic artist, ENAC, Toulouse France

- **Jérôme Espenan**

Graphic artist, ENAC, Toulouse France

NOTES



Organizers



Funding sponsor



Scientific sponsors



Sponsors



Geometric GSI'19 Science of Information

Toulouse, 27th - 29th August 2019

// GSI'19 Social Events

Welcome reception

The welcome reception will take place on **Tuesday, August 27th** in Amphi Bellonte's Hall at ENAC from **19:00 to 20:00**.

The cocktail will consist of:

- Tasting of 5 wines discovered in agreement with the Mont Royal cheeses
- Animation and service delivery of sommelier
- Oenology service and advice to the guests
- Technical sheets of all the wines presented

Gala dinner

The gala dinner will be held on **Wednesday, August 28th** in Salle des Colonnes at Hôtel-Dieu Saint-Jacques from **20:00 to 22:00**.

Shuttle bus will be provided from ENAC to Hôtel-Dieu Saint-Jacques at **19:15**.

The return trip is not provided.

The dinner is included in the **FULL registrations only**.