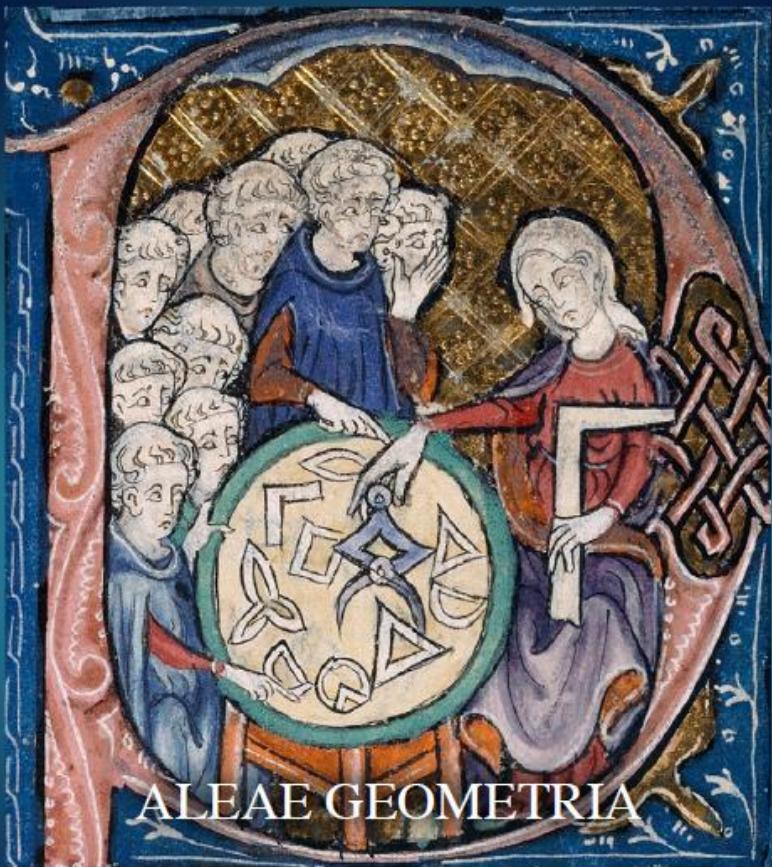


**5<sup>th</sup> Conference on  
the Geometric Science  
of Information**

**GSI'21**

**Sorbonne University**

**July 21<sup>st</sup> - 22<sup>nd</sup> - 23<sup>rd</sup> 2021**



PARIS

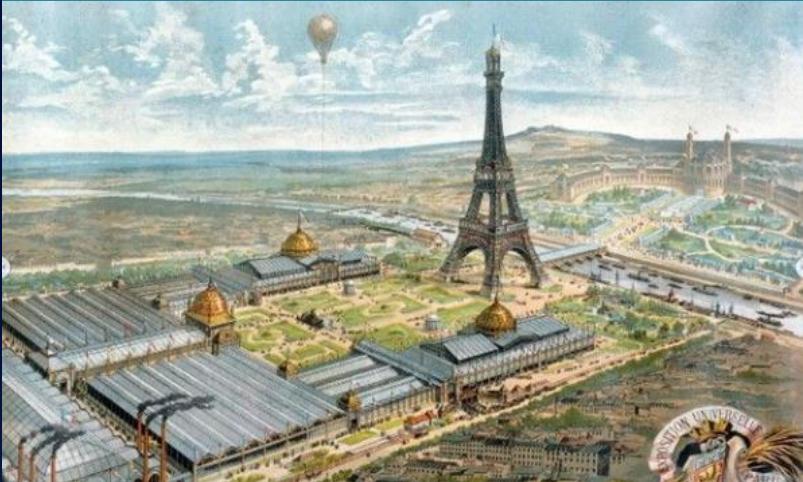


SCAI  
SORBONNE CENTER FOR  
ARTIFICIAL INTELLIGENCE



**GSI'21**

**LEARNING GEOMETRIC STRUCTURES**



# Geometric Science of Information

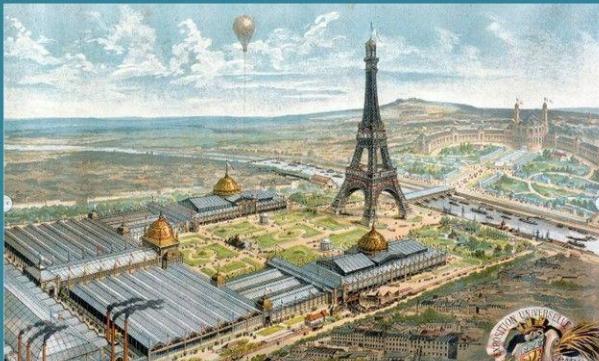
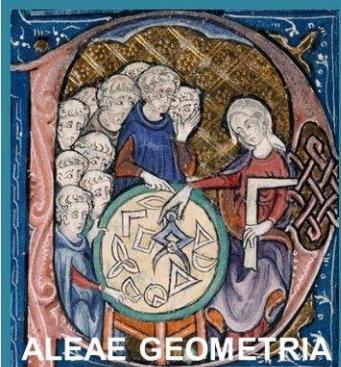
## *GSI'21 Conference, 21<sup>st</sup>-23<sup>rd</sup> July 2021*

## *Sorbonne University, Paris*

General Chairs: Frédéric BARBARESCO\*, Frank NIELSEN\*\*

(\*) THALES, SEE ISIC Club President (Ingénierie des Systèmes d'Information & Communications)

(\*\*) Sony CSL Tokyo



**GSI'21**  
LEARNING GEOMETRIC STRUCTURES

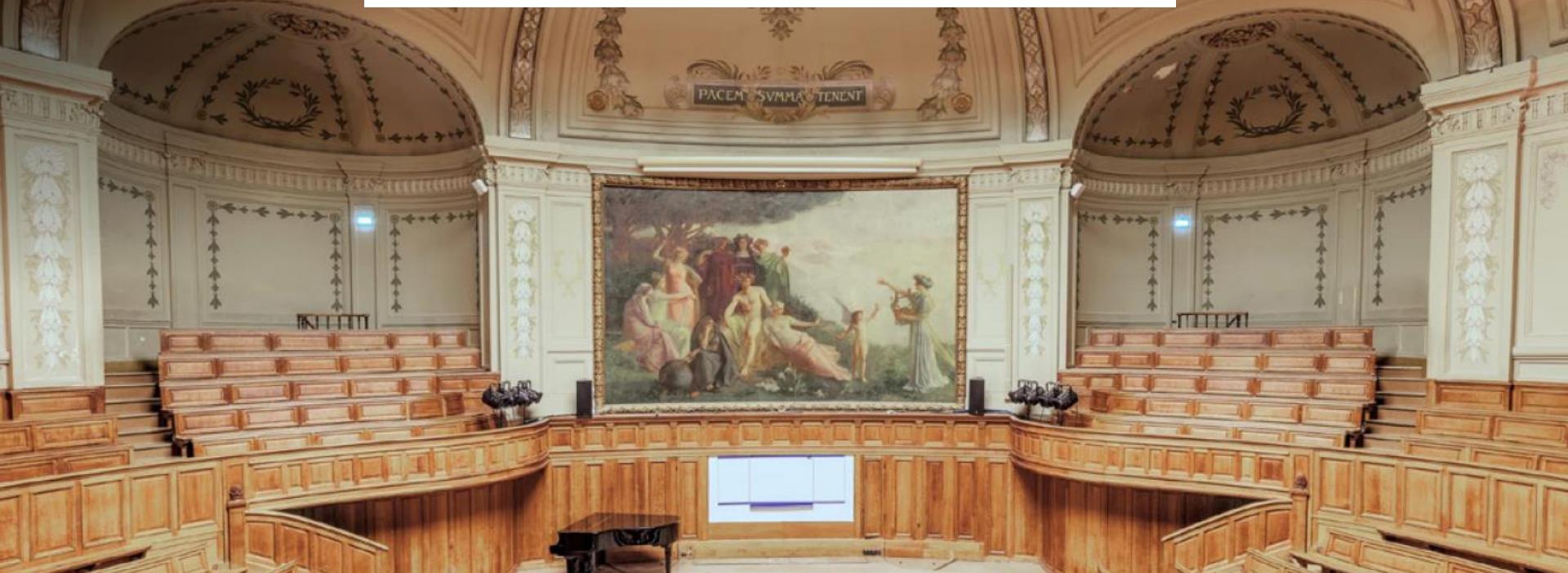


# 5<sup>th</sup> GSI'21 Conference

# LEARNING GEOMETRIC STRUCTURES

**Sorbonne University**

July 21<sup>st</sup> – 22<sup>nd</sup> – 23<sup>rd</sup> 2021



SORBONNE UNIVERSITY SINCE 1257

# GSI'21 Sponsors



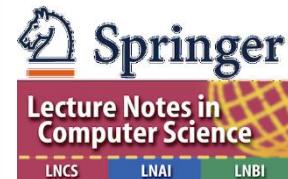
## Sponsors



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# GSI'21 Editors & Organizer

## | Editors



Information  
Geometry

## | Organizer



# SEE at a glance

- Meeting place for science, industry and society
- An officially recognised non-profit organisation
- About 2000 members and 5000 individuals involved
- Large participation from industry (~50%)
- 19 «Clubs techniques» and 12 «Groupes régionaux»
- Organizes conferences and seminars
- Initiates/attracts International Conferences in France
- Institutional French member of IFAC and IFIP
- Awards (Glavieux/Brillouin Prize, Général Ferrié Prize, Néel Prize, Jerphagnon Prize, Blanc-Lapierre Prize, Thévenin Prize), grades and medals (Blondel, Ampère)
- Publishes 3 periodical publications (REE, ...) & 3 monographs each year
- Web: <http://www.see.asso.fr> and LinkedIn SEE group
- SEE Presidents: Louis de Broglie, Paul Langevin, ...



Fédération Française  
de Sociétés Scientifiques



Paul Langevin



Louis de Broglie

1883-2021: From SIE & SFE to SEE: 138 years of Sciences

Société de l'électricité, de l'électronique et des technologies de l'information et de la communication



1881

Exposition Internationale d'Electricité



1883: SIE  
Société  
Internationale  
des Electriciens

1886: SFE  
Société  
Française  
des Electriciens



2013: SEE  
17 rue de l'Amiral Hamelin  
75783 Paris Cedex 16



[scai.sorbonne-universite.fr](http://scai.sorbonne-universite.fr)



# Facts & Figures and Actions

**10** Scientists

**20** Industrial partners

**150** Doctoral and postdoctoral researchers

**8** Academic partners

**30** Master's students trained each year

**5** Master's degrees + lifelong learning programs

**2** Locations in Paris and Abu Dhabi

**1** Chairs of excellence

**5** Funding opportunities

**2** Interdisciplinary projects

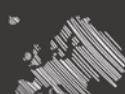
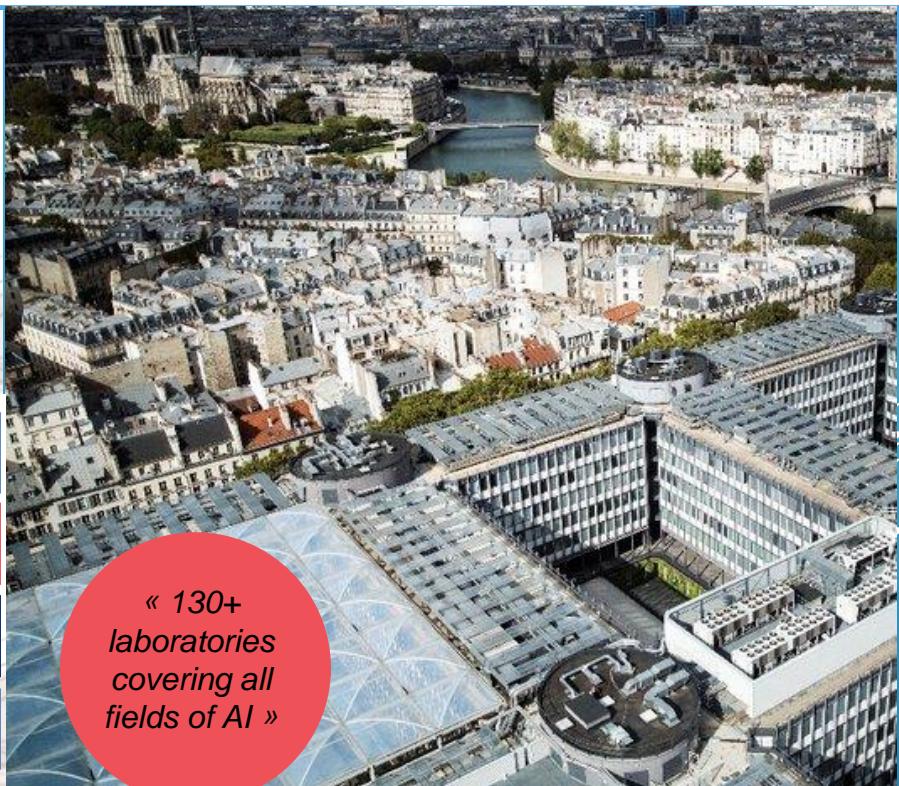
**6** Doctoral program

**3** Collaborative responses to scientific calls

**7** Scientific events

**4** Industrial partnerships

**8** Innovative training programs



ellis  
unit | PARIS

SEPTEMBER 15<sup>TH</sup>  
2020

# GSI'21 Figures

- **GSI'21 Registrations:** 180 participants (2/3 In person)
- **GSI'21 Program**
  - 6 Keynote Speakers
  - 101 talks in 22 Oral sessions
  - 13 Posters in 2 Poster sessions
- **Papers Acceptance Rate of 83% based on 389 reviews (minimum of 2 reviews per paper)**
- **Papers Origin:** France 36%, Germany 11%, USA 10%, Japan 10%, UK 4%, Denmark 3%, Netherlands 3%, Russia 2%, Poland 2%
- **But also from:** Australia, Austria, Belgium, Canada, Chile, China, Croatia, Czechia, India, Mexico, Nigeria, Norway, Portugal, Romania, Spain, Sweden, Switzerland, United Arab Emirates, Venezuela, Viet Nam

# 22 GSI'21 Oral Sessions

- **Probability & statistics on Riemannian Manifolds** - Chairs: Xavier Pennec, Cyrus Mostajeran
- **Shapes Spaces** - Chairs: Salem Said, Joan Glaunès
- **Geometric and structure preserving discretizations** - Chairs: Alessandro Bravetti, Manuel de Leon
- **Lie Group Machine Learning** - Chairs: Frédéric Barbaresco, Gery de Saxcé
- **Harmonic Analysis on Lie Groups** - Chairs: Jean-Pierre Gazeau, Frédéric Barbaresco
- **Geometric Mechanics** - Chairs: Gery de Saxcé, Frédéric Barbaresco
- **Sub-Riemannian Geometry and Neuromathematics** - Chairs: Alessandro Sarti, Dario Prandi
- **Statistical Manifold & Hessian Information Geometry** - Chairs: Noémie Combe, Michel Nguiffo Boyom
- **Information Geometry in Physics** - Chairs: Geert Verdoolaege, Jun Zhang
- **Geometric & Symplectic Methods for Hydrodynamical Models** - Chairs: Cesare Tronci, François Gay-Balmaz
- **Geometry of Quantum States** - Chairs: Florio Maria Ciaglia, Michel Berthier
- **Deformed entropy, cross-entropy, and relative entropy** - Chairs: Ting-Kam Leonard Wong, Léonard Monsaingeon
- **Geometric structures in thermodynamics and statistical physics** - Chairs: Hiroaki Yoshimura, François Gay-Balmaz
- **Geometric Deep Learning** - Chairs: Gabriel Peyré, Erik J. Bekkers
- **Computational Information Geometry 1** - Chairs: Frank Nielsen, Clément Gauchy
- **Computational Information Geometry 2** - Chairs: Giovanni Pistone, Goffredo Chirco
- **Optimal Transport & Learning** - Chairs: Yaël Frégier, Nicolas Garcia Trillos
- **Statistics, Information and Topology** - Chairs: Pierre Baudot, Michel Nguiffo Boyom
- **Topological and Geometrical Structures in Neurosciences** - Chairs: Pierre Baudot, Giovani Petri
- **Manifolds & Optimization** - Chairs: Stéphanie Jehan-Besson, Bin Gao
- **Divergence Statistics** - Chairs: Michel Broniatowski, Wolfgang Stummer
- **Transport information geometry** - Chairs: Wuchen Li, Philippe Jacquet

# GSI'21 keynote speakers

Michel Broniatowsk



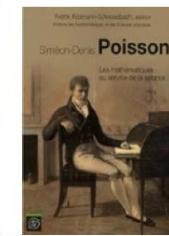
TITLE: SOME INSIGHTS ON STATISTICAL DIVERGENCES AND CHOICE OF MODELS

Maurice de Gosson



TITLE: GAUSSIAN STATES FROM A SYMPLECTIC GEOMETRY POINT OF VIEW

Yvette Kosmann-Schwarzbach



TITLE: STRUCTURES OF POISSON GEOMETRY: OLD AND NEW

Giuseppe Longo



TITLE: USE AND ABUSE OF «DIGITAL INFORMATION» IN LIFE SCIENCES, IS GEOMETRY OF INFORMATION A WAY OUT?

Jean Petitot



TITLE : THE PRIMARY VISUAL CORTEX AS A CARTAN ENGINE

Max Welling



TITLE: EXPLORING QUANTUM STATISTICS FOR MACHINE LEARNING

**GSI'21 OPENING DAY, July 21<sup>st</sup>**  
**Amphi RICHELIEU – Sorbonne Université**



08.30-09.00	Welcome Desk	
09.00-09.30	<b>Opening by GSI'21 chairs &amp; sponsors</b> Frédéric BARBARESCO & Frank NIELSEN	
09.30-10.30	<b>Use and abuse of «digital information» in life sciences, is Geometry of Information a way out?</b> Giuseppe LONGO - Centre Cavaillès, CNRS & Ens Paris and School of Medicine, Tufts University, Boston	Keynote chair: Frédéric BARBARESCO
10.30-11.00	<b>Coffee Break + Group Photo</b>	
11.00-12.00	<b>Structures of Poisson geometry: old and new</b> Yvette KOSMANN-SCHWARZBACH - Professeur des universités honoraire - France	Keynote chair: Gery de SAXCE
12.00-14.00	<b>Lunch Break (not included) + mini-hackathon</b> GEOMSTATS at SCAI	Mini-hackathon: Nina Miolane
14.00-15.00	<b>Some insights on statistical divergences and choice of models</b> Michel BRONIATOWSKI - Sorbonne Université, France	Keynote Chair: Frank NIELSEN
15.00-16.00	<b>Gaussian states from a symplectic geometry point of view</b> Maurice de GOSSON - Faculty of Mathematics, NuHAG group - University of Vienna, Austria	Keynote Chair: Jean-Pierre GAZEAU
16.00-16.30	<b>Coffee Break</b>	
16.30-17.30	<b>Exploring quantum statistics for machine learning</b> Max WELLING, Informatics Institute, University of Amsterdam and Qualcomm Technologies	Keynote Chair: Gabriel PEYRE
17.30-18.30	<b>The primary visual cortex as a Cartan engine</b> Jean PETITOT - CAMS, EHESS (Ecole des Hautes Etudes en Sciences Sociales)	Keynote Chair: Daniel BENNEQUIN
20.00-21.30	<b>Welcome cocktail - SCAI Sorbonne (Jussieu site)</b>	

**PLENARY SESSION JULY 21<sup>ST</sup>**

Amphithéâtre Richelieu  
 Paris-Sorbonne University  
 17 rue de la Sorbonne, 75005 Paris

	SESSIONS ROOM A	SESSIONS ROOM B
08.30-10.10	<b>Probability &amp; statistics on Riemannian Manifolds</b> Chairs: Xavier Pennec, Cyrus Mostajeran	<b>Sub-Riemannian Geometry and Neuromathematics</b> Chairs: Alessandro Sarti, Dario Prandi
10.10-10.30	<b>Coffee Break + Posters Session 1</b>	
10.30-12.10	<b>Shapes Spaces</b> Chairs: Salem Said, Joan Glaunès	<b>Geometry of Quantum States</b> Chairs: Florio Maria Ciaglia, Michel Berthier
12.10-13.10	<b>Lunch Break (not included) + mini-hackathon GEOMSTATS at SCAI</b>	
13.10-14.30	<b>Geometric and structure preserving discretizations</b> Chairs: Alessandro Bravetti, Manuel de Leon	<b>Information Geometry in Physics</b> Chairs: Geert Verdoolaege, Jun Zhang
14.30-16.10	<b>Lie Group Machine Learning</b> Chairs: Frédéric Barbaresco, Gery de Saxcé	<b>Geometric &amp; Symplectic Methods for Hydrodynamical Models</b> Chairs: Cesare Tronci, François Gay-Balmaz
16.10-16.30	<b>Coffee Break + Posters Session 1</b>	
16.30-18.10	<b>Harmonic Analysis on Lie Groups</b> Chairs: Jean-Pierre Gazeau, Frédéric Barbaresco	<b>Statistical Manifold &amp; Hessian Information Geometry</b> Chairs: Noémie Combe, Michel Nguiffo Boyom
18.10-19.30	<b>Geometric Mechanics</b> Chairs: Gery de Saxcé, Frédéric Barbaresco	<b>Deformed entropy, cross-entropy, and relative entropy</b> Chairs: Ting-Kam Leonard Wong, Léonard Monsaingeon
20.00-22.30	<b>GALA Dinner</b>	

## SESSIONS JULY 22<sup>ND</sup>

Centre International  
 de Conférences  
 Sorbonne Université  
 Patio 44, 4 place Jussieu 55,  
 75005 Paris

**GSI'21, July 23<sup>rd</sup>**  
**Jussieu site - Sorbonne Université**

	SESSIONS ROOM A	SESSIONS ROOM B
08.30-09.50	<b>Transport information geometry</b> Chairs: Wuchen Li, Philippe Jacquet	<b>Statistics, Information and Topology</b> Chairs: Pierre Baudot, Michel Nguiffo Boyom
09.50-10.10	<b>Coffee Break + Posters Session 2</b>	
10.10-11.50	<b>Geometric Deep Learning</b> Chairs: Gabriel Peyré, Erick J. Bekkers	<b>Topological and Geometrical Structures in Neurosciences</b> Chairs: Pierre Baudot, Giovani Petri
11.50-13.00	<b>Lunch Break (not included) + mini-hackathon GEOMSTATS at SCAI</b>	
13.00-14.40	<b>Computational Information Geometry 1</b> Chairs: Frank Nielsen, Clément Gauchy	<b>Manifolds &amp; Optimization</b> Chairs: Stéphanie Jehan-Besson, Bin Gao
14.40-16.20	<b>Computational Information Geometry 2</b> Chairs: Giovanni Pistone, Goffredo Chirco	<b>Divergence Statistics</b> Chairs: Michel Broniatowski, Wolfgang Stummer
16.20-16.40	<b>Coffee Break + Posters Session 2</b>	
16.40-18.00	<b>Optimal Transport &amp; Learning</b> Chairs: Yaël Frégier, Nicolas Garcia Trillos	<b>Geometric structures in thermodynamics and statistical physics</b> Chairs: Hiroaki Yoshimura, François Gay-Balmaz
18.00-18.30	<b>GSI'21 Closure</b>	

## SESSIONS JULY 23<sup>RD</sup>

Centre International  
de Conférences  
Sorbonne Université  
Patio 44, 4 place Jussieu 55,  
75005 Paris

## Poster Session 1

Clustering Schemes on the Torus with Application to RNA Clashes	Henrik Wiechers, Benjamin Eltzner, Stephan F. Huckemann and Kanti V. Mardi
Separating Point Patterns for Fingerprints	Johannes Wieditz, Yvo Pokern, Dominic Schuhmacher, and Stephan Huckemann
Statistical manifolds & Hidden symmetries	Noemie Combe, Philippe Combe, and Hanna Nencka
Nonparametric Estimation of Probability Density Functions with Gaussian Processes	T. T. Tran, C. Samir, and A. Fradi
On Information (pseudo) Metric	Pierre Baudot
Nonparametric Regression on the Compact Stiefel Manifold	Ines Adouani and Chafik Samir
On Riemannian Stochastic Approximation Schemes with Fixed Step-Size	Alain Durmus, Pablo Jiménez, Éric Moulines, Salem Said

## Poster Session 2

Application of information geometry in uncertainty quantification of computer codes	Clément Gauchy and Jérôme Stenger
On Geometry of Stein Goodness-of-fit Testing	Wenkai Xu and Takeru Matsuda
Simulation of Quantum Noise for Coherent States	Marco Frasca and Alfonso Farina
On the Quantization of the Uniform Source on the n-Sphere ( $n = 1, 2, 3$ )	Stéphane Ragot and Mohamed Yaoumi
Flattening Multiparameter Hierarchical Clustering Functors	Dan Shiebler
Learning the GENERIC evolution	Martin Sípká and Michal Pavelka

## | Poster Sessions

# Computational Geometric Science of Information: GEOMSTATS HACKATHON

If you are interested in:

- ✓ using and understanding existing implementations of differential geometry,
- ✓ implementing ideas, examples, experiments for a/your research paper,
- ✓ adding hands-on exercises or examples to your differential geometry classes,
  - ✓ learning how to code differential geometric structures,
  - ✓ learning how to contribute to an open-source GitHub project,

Come to the hackathon!

**When?**

21<sup>st</sup>, 22<sup>nd</sup> & 23<sup>rd</sup> of July from 12.30 pm to 2.30 pm (lunch break)

**Where?**

Esclangon building at SCAI, 2<sup>nd</sup> floor / (see map 44)

**How?**

Metro: Jussieu (lines 7 and 10)

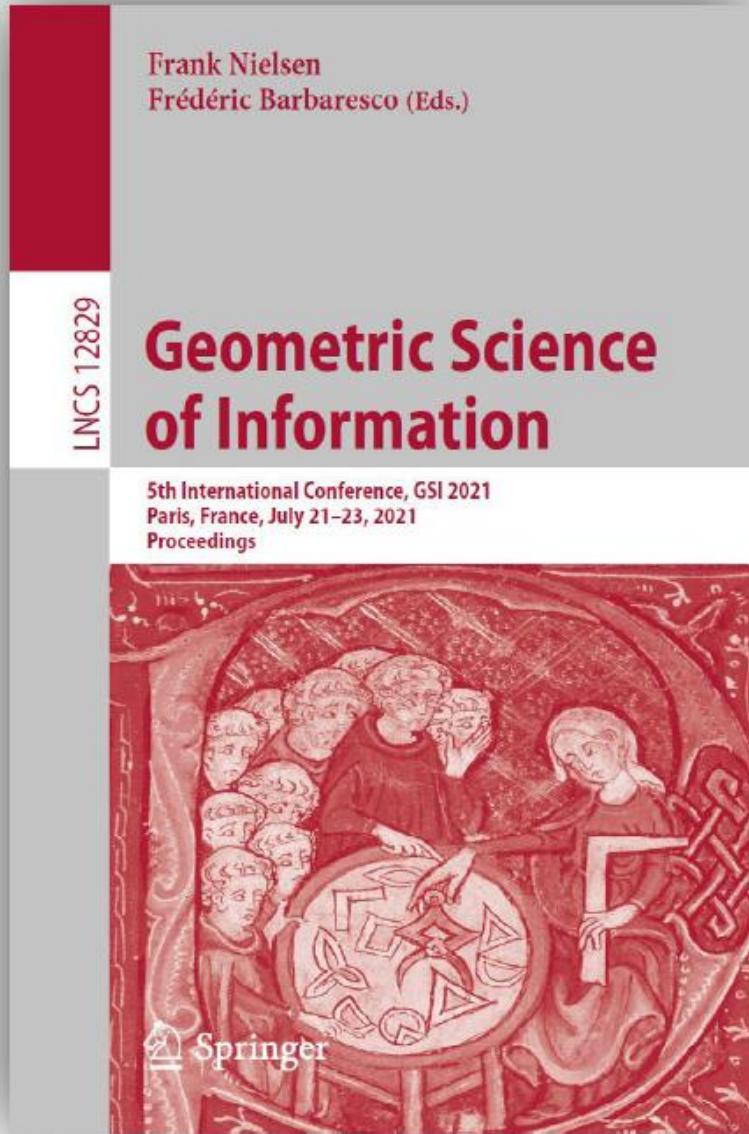
**Nicolas Guigui** and **Nina Miolane** will be available to answer your questions about computational geometric science of information, guide you through existing implementations, and help you translate your ideas into code. Feel free to join any day, for any duration, with or without a computational project in mind, with or without coding experience!



Nina Miolane



Nicolas Guigui



<https://www.springer.com/gp/book/9783030802080>

<https://franknielsen.github.io/GSI/>

# GSI'21 Papers Awards



GSI 2021

July 21<sup>st</sup>-23<sup>rd</sup> 2021  
Paris-Sorbonne University



The 5<sup>th</sup> international conference on  
Geometric Science of Information  
Learning geometric structures

## Best Paper Award

PRESENTED TO

First1 Last1 and First2 Last2.....

AUTHOR(S) OF THE PAPER

Long title.....

Frédéric Barbaresco  
Thales, France  
General co-chair

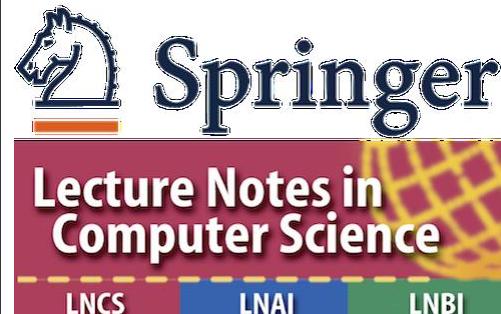
Frank Nielsen  
Sony Computer Science Laboratories, Japan  
General co-chair

Gabriel Peyré  
CNRS, ENS, France  
Prize committee, President



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GSI 2021

July 21<sup>st</sup>-23<sup>rd</sup> 2021  
Paris-Sorbonne University



The 5<sup>th</sup> international conference on  
Geometric Science of Information

Learning geometric structures

## WiMlds Best Paper Award

PRESENTED TO

First1 Last1 and First2 Last2.....

AUTHOR(S) OF THE PAPER

Long title.....

Frédéric Barbaresco  
Thales, France  
General co-chair

Frank Nielsen  
Sony Computer Science Laboratories, Japan  
General co-chair



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...  
WiMlds  
Jury President

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### CONFERENCE CO-CHAIRS:

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**Frédéric Barbaresco**, President of SEE ISIC Club, THALES KTD PCC SENSING SEGMENT LEADER, Meudon, France

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**Xavier Fresquet**, Sorbonne Center for AI (SCAI) Deputy Director

**Michel Broniatowski**, Sorbonne University Professor (LPSM lab)

**Jean-Pierre Francoise**, Sorbonne University Professor (JLL lab)

**Olivier Schwander**, Sorbonne University Professor (LIP6 lab)

**Gabriel Peyré**, ELLIS Paris Unit & ENS Paris

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**Jesus Angulo**, Mines ParisTech

**Marc Arnaudon**, Bordeaux University

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**Roger Balian**, CEA  
**Frédéric Barbaresco**, Thales Land & Air Systems  
**Pierre Baudot**, Median Technologies  
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**Yannick Berthoumieu**, Bordeaux University  
**Jeremie Bigot**, Bordeaux University  
**Silvere Bonnabel**, Mines ParisTech  
**Michel Boyom**, Montpellier University  
**Marius Buliga**, Simion Stoilow Institute of Mathematics of the Romanian Academy  
**Giovanna Citi**, Bologna University, Italy  
**Laurent Cohen**, Paris Dauphine University  
**Nicolas Couellan**, ENAC  
**Ana Bela Cruzeiro**, Universidade de Lisboa  
**Remco Duits**, Eindhoven University of Technology  
**Stanley Durleman**, INRIA  
**Stephane Puechmorel**, ENAC Toulouse  
**Fabrice Gamboa**, Institut Mathématique de Toulouse  
**Jean-Pierre Gazeau**, Paris Diderot University  
**François Gay-Balmaz**, ENS Ulm  
**Mark Girolami**, Imperial College London  
**Audrey Giremus**, Bordeaux University  
**Hatem Hajri**, IRT SystemX  
**Susan Holmes**, Stanford University  
**Stephan Huckeman**, Goettingen University, Germany  
**Jean Lerbet**, Evry University  
**Nicolas Le Bihan**, Grenoble University  
**Alice Le Brigant**, Pantheon Sorbonne University

**Luigi Malago**, Romanian Institute of Science and Technology  
**Jonathan Manton**, The University of Melbourne  
**Gaetan Marceau-Caron**, MILA R&D and Tech Transfer  
**Matilde Marcolli**, CALTECH  
**Jean-François Marcotorchino**, Sorbonne University  
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**Antonio Mucherino**, IRISA, University of Rennes 1  
**Florence Nicol**, ENAC  
**Frank Nielsen**, Ecole Polytechnique, Paris-Saclay University  
**Richard Nock**, Université Antilles-Guyane  
**Yann Ollivier**, FACEBOOK FAIR Paris  
**Steve Oudot**, INRIA  
**Pierre Pansu**, Paris-Saclay University  
**Xavier Pennec**, INRIA  
**Giovanni Pistone**, de Castro Statistics, Collegio Carlo Alberto  
**Oliver Rioul**, Telecom ParisTech  
**Gery de Saxcé**, Lille University  
**Salem Said**, Bordeaux University  
**Alessandro Sarti**, EHESS Paris  
**Rodolphe Sepulchre**, Liège University  
**Oliver Schwander**, Sorbonne University  
**Stefan Sommer**, Copenhagen University  
**Dominique Spehner**, Grenoble University  
**Wolfgang Stummer**, Friedrich-Alexander-Universität Erlangen, Nürnberg

**Alain Trouvé**, Ecole Normale Supérieure Paris-Saclay  
**Geert Verdoncklaege**, Ghent University  
**Rene Vidal**, Johns Hopkins University  
**Jun Zhang**, University of Michigan, Ann Arbor  
**Pierre-Antoine Absil**, Université Catholique de Louvain  
**Florio M. Ciaglia**, Max Planck Institute for Mathematics in the Sciences  
**Manuel de Leon**, Real Academia de Ciencias, Madrid  
**Goffredo Chirco**, Theoretical Physics Group INFN Naples  
**Höng Van Lê**, Institute of Mathematics f Czech Academy of Sciences  
**Bruno Iannazzo**, Università degli Studi di Perugia  
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**Ana Bela Ferreira Cruzeiro**, Universidade de Lisboa  
**Nina Miolane**, Stanford University  
**Cyrus Mostajeran**, Cambridge University  
**Rodolphe Sepulchre**, Cambridge University  
**Stefan Horst Sommer**, University of Copenhagen  
**Wolfgang Stummer**, University of Erlangen-Nürnberg  
**Zdravko Terze**, University of Zagreb  
**Geert Verdoncklaege**, Ghent University  
**Wuchen Li**, UCLA  
**Hiroaki Yoshimura**, Waseda University  
**Jean Claude Zambrini**, Universidade de Lisboa  
**Jun Zhang**, University of Michigan



# | Event Map for GSI'21

 **Sessions Locations**  
July 21<sup>st</sup>  
Amphithéâtre Richelieu  
Paris-Sorbonne University  
17 rue de la Sorbonne, 75005 Paris

 **July 22<sup>nd</sup> & 23<sup>rd</sup>**  
Centre International  
de Conférences  
Sorbonne Université  
Patio 44, 4 place Jussieu 55,  
75005 Paris

 **Welcome Cocktail**  
July 21<sup>st</sup>  
SCAI  
Sorbonne Université  
Campus Pierre et Marie Curie 4  
Place Jussieu, 75005 Paris

 **Gala Dinner Boat Cruise**  
July 22<sup>nd</sup>  
Boarding Point Canauxrama  
Port de l'Arsenal  
Port on Canal Saint-Martin facing  
50 bd de la Bastille, 75012 Paris

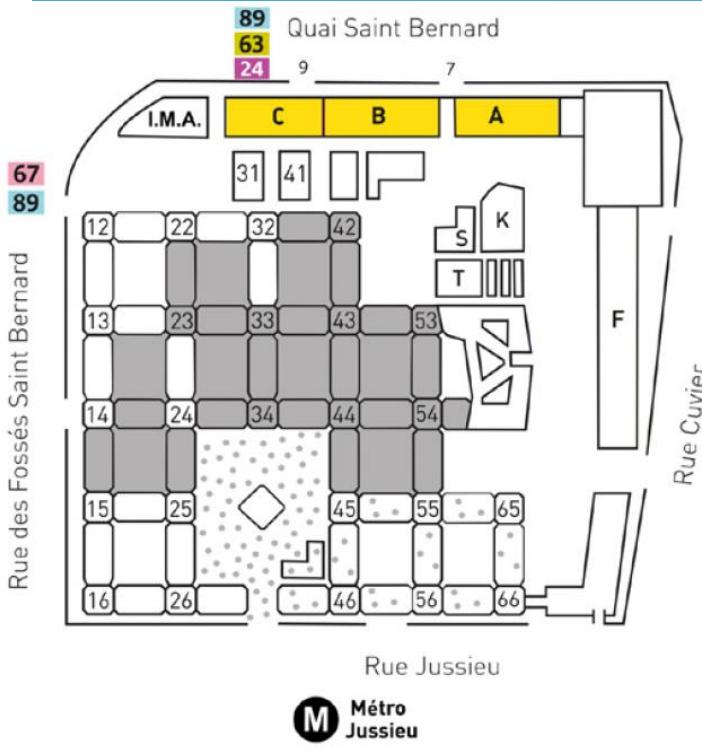


#### Comment venir à scai ?

Longer l'allée à droite de l'entrée principale jusqu'au dernier bâtiment « Esclangon ».

A l'intérieur du bâtiment Esclangon, prenez la 1<sup>re</sup> sortie à gauche puis l'escalier à droite. scai se situe en haut des marches.

## | Location Information





## Welcome Cocktail

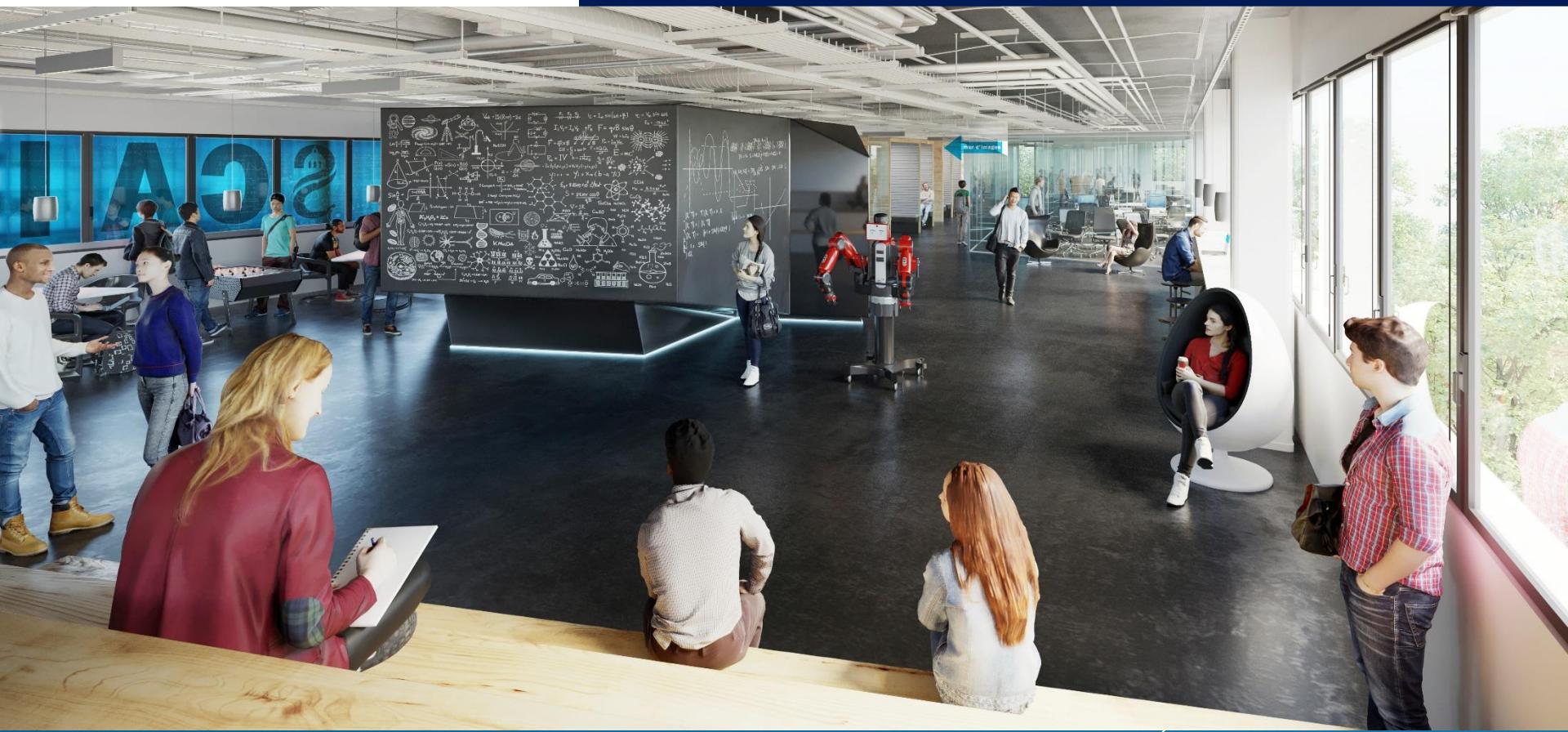
July 21<sup>st</sup>

SCAI

Sorbonne Université  
Campus Pierre et Marie Curie 4  
Place Jussieu, 75005 Paris



# SCAI@Paris

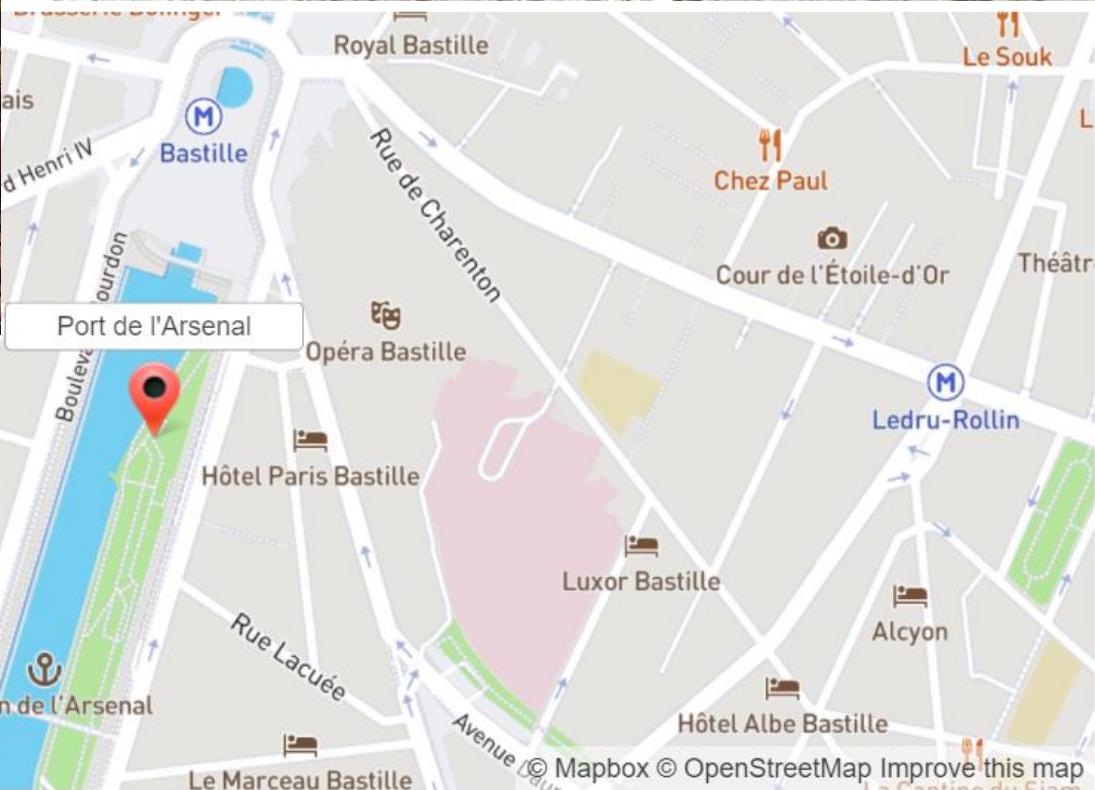




## X Gala Dinner Boat Cruise

July 22<sup>nd</sup>

Boarding Point Canauxrama  
Port de l'Arsenal  
Port on Canal Saint-Martin facing  
50 bd de la Bastille, 75012 Paris





**Looking for a restaurant nearby  
La Sorbonne or Jussieu site?**

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July 21<sup>st</sup>  
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A personal code for each attendee will be available  
at the Registration Desk.

Feel free to ask for one!



## Welcome message from Frank Nielsen

**Sony Computer Science Laboratories Inc.  
GSI 2021 co-chair**



# Sony CSL



# GSI biannual conferences

<https://franknielsen.github.io/GSI/>



GSI'13 Mines ParisTech



GSI'15 Ecole Polytechnique



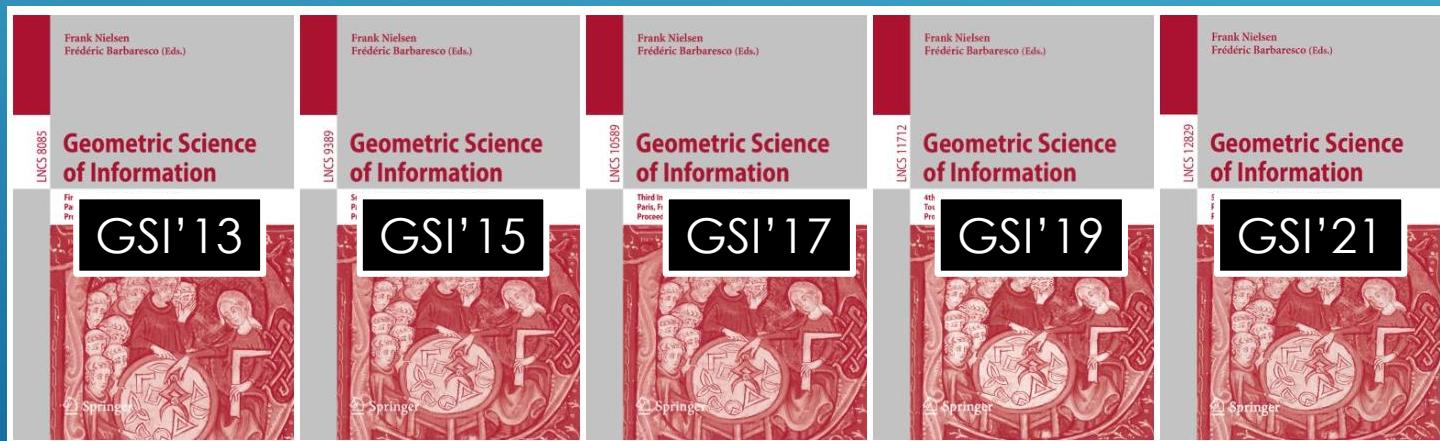
GSI'17 Mines ParisTech



GSI'19 ENAC

# GSI LNCS Proceedings

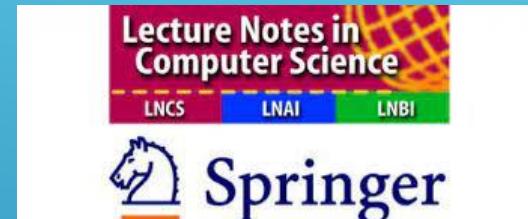
## About 500 papers in about 4500 pages



<https://link.springer.com/conference/gsi>

# GSI 2021: Two awards with prizes

- “**GSI21 Best paper**” sponsored by Springer (1000€)
- “**WiMLDS GSI21 Best paper**” sponsored by MDPI Mathematics (1000€)



<https://franknielsen.github.io/GSI/>



TGSI'17 CIRM



FGSI'19 IMAG

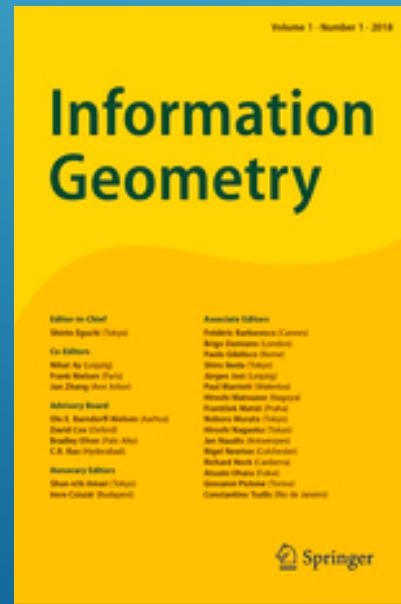
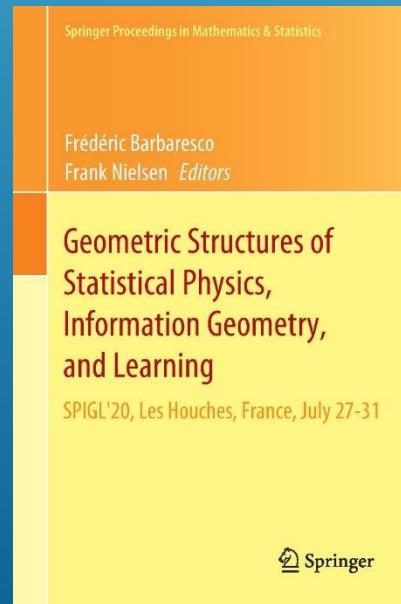
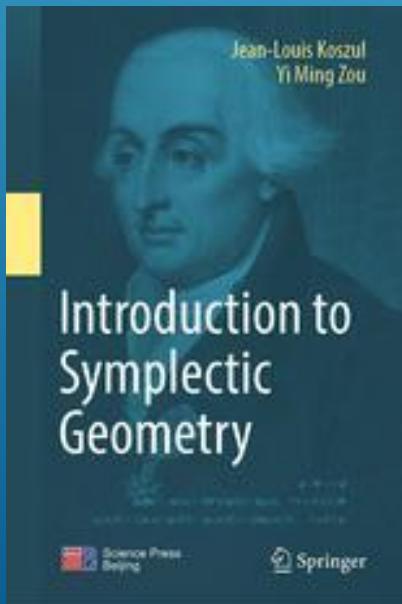
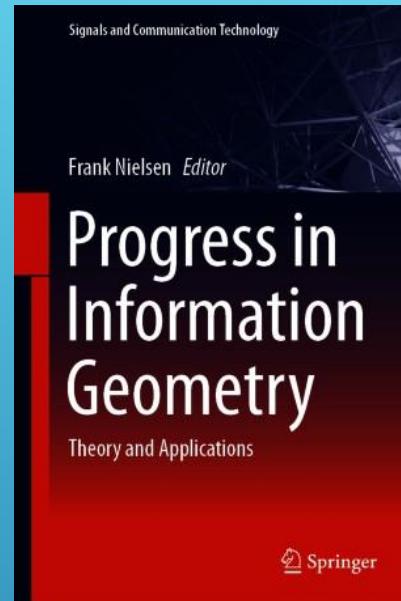
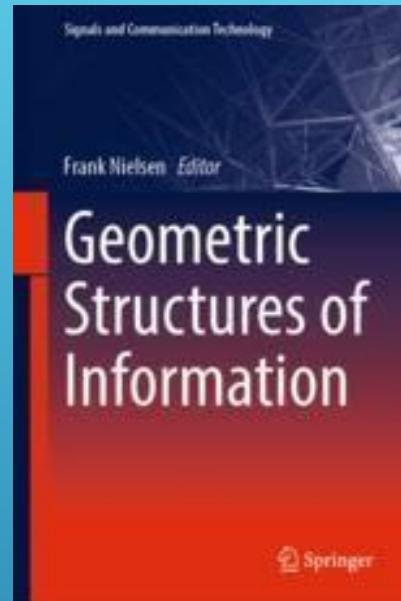
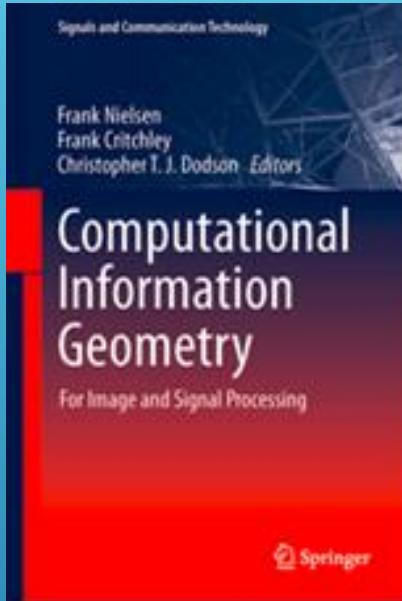


GRETSI'19 PEYRESQ



SPIGL'20 LES HOCHES

# Last Publications

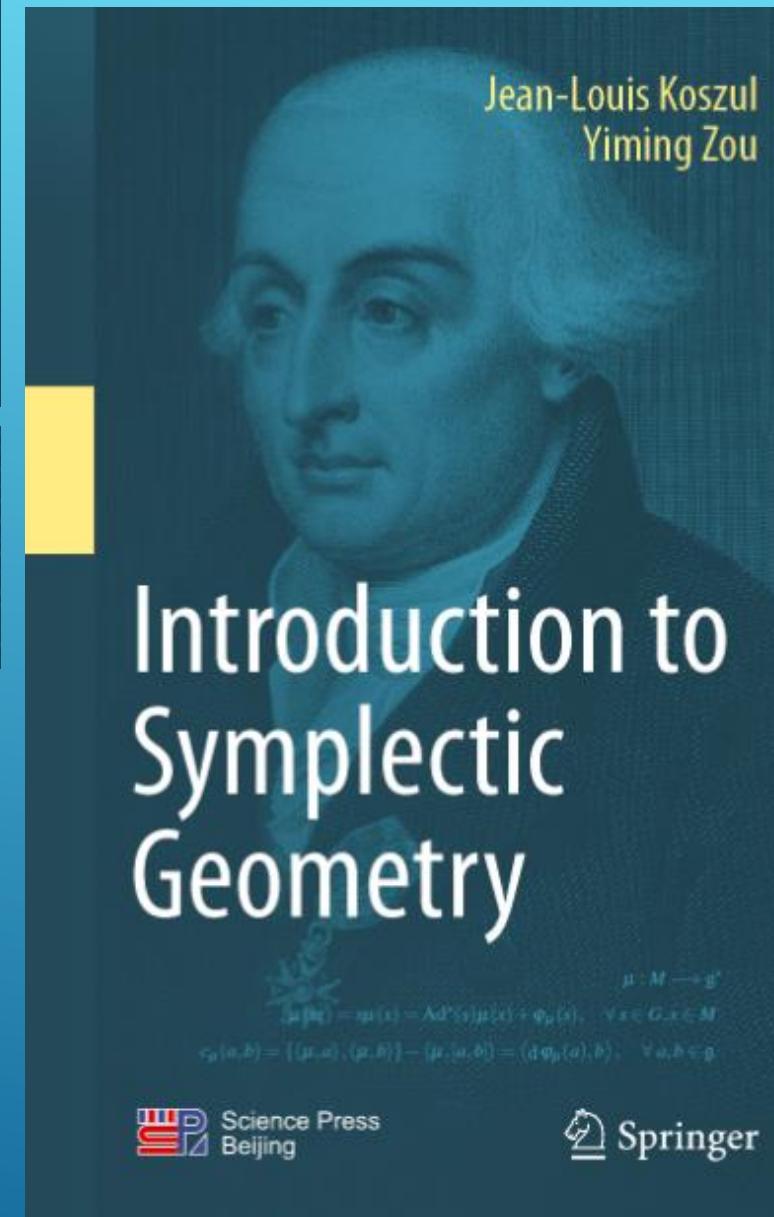


Jean-Louis Koszul · Yiming Zou

## Introduction to Symplectic Geometry

Forewords by Michel Nguliffo Boyom, Frédéric Barbaresco and Charles-Michel Marle

This introductory book offers a unique and unified overview of symplectic geometry, highlighting the differential properties of symplectic manifolds. It consists of six chapters: Some Algebra Basics, Symplectic Manifolds, Cotangent Bundles, Symplectic G-spaces, Poisson Manifolds, and A Graded Case, concluding with a discussion of the differential properties of graded symplectic manifolds of dimensions (o,n). It is a useful reference resource for students and researchers interested in geometry, group theory, analysis and differential equations.



# ECOLE DE PHYSIQUE DES HOUCHESSPIGL'20, JULY 2020



Springer Proceedings in Mathematics & Statistics

Frédéric Barbaresco  
Frank Nielsen *Editors*

## Geometric Structures of Statistical Physics, Information Geometry, and Learning

SPIGL'20, Les Houches, France, July 27-31

Springer

SPIGL'20    cnrs    cea    UNIVERSITÉ DE LYON  
Grenoble INP | UGA    UGA Université Grenoble Alpes

Joint Structures and Common Foundation of Statistical Physics, Information Geometry and Inference for Learning  
26<sup>th</sup> July to 31<sup>st</sup> July 2020

Carnot    Clausius    Massieu    Helmholtz    Boltzmann    Gibbs    Duhem  
Poincaré    Planck    Langevin    Carathéodory    Brillouin    Callen    Balian  
Fréchet    Fisher    CLAUDE E. SHANNON    Cramer    Rao    Chentsov    Amari  
Bargmann    Souriau    Kirillov    Kostant    Koszul    Vinberg  
Noether    Libermann    Choquet-Bruhat    DeWitt-Morette    Uhlenbeck    Vergne    Saint-Raymond

<https://www.springer.com/jp/book/9783030779566>

<https://www.youtube.com/playlist?list=PLo9ufcrEqwWEtBPgQPJwAJhoUCHMbR>  
Or

<https://franknielsen.github.io/SPIG-LesHouches2020/>

# SPRINGER « INFORMATION GEOMETRY » Journal Volume 4, Issue 1, July 2021

<https://link.springer.com/journal/41884/volumes-and-issues/4-1>

Information Geometry (2021) 4:155–157  
<https://doi.org/10.1007/s41884-020-00042-2>

## PREFACE

### Special Issue “Affine differential geometry and Hesse geometry: a tribute and memorial to Jean-Louis Koszul”

Hideyuki Ishi<sup>1</sup>

Published online: 7 January 2021  
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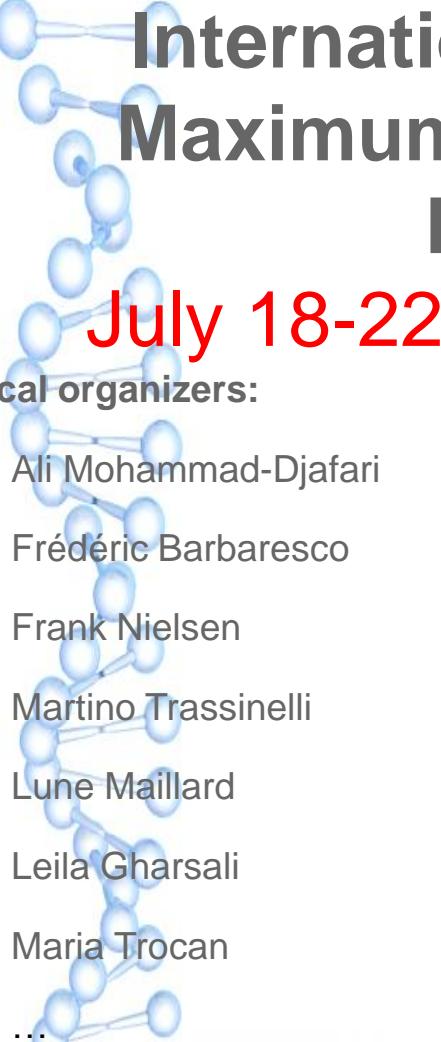


<https://link.springer.com/article/10.1007/s41884-020-00042-2>

# Information Geometry

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 Springer



# International Conference on Bayesian and Maximum Entropy methods in Science and Engineering, MaxEnt 2022

July 18-22 2022, Institut Henri Poincaré, Paris

## Local organizers:

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- Frédéric Barbaresco
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- Robert Niven
- Wolfgang von der Linden
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- Romke Bontekoe
- Barrie Stokes
- Donald Nelson
- Keith Earle
- ...

## Institutions & Locals

Institut Henri Poincaré

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Institut des NanoSciences de Paris (INSP)

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Institut des Neurosciences Paris-Saclay

<https://neuropsi.cnrs.fr/>

Sorbonne Center for Artificial Intelligence  
(SCAI)

<https://scai.sorbonne-universite.fr/>

Institut DATAIA

<https://dataia.eu/en>

Artificial Intelligence in Paris and Europe

<https://ellis-paris.github.io/>

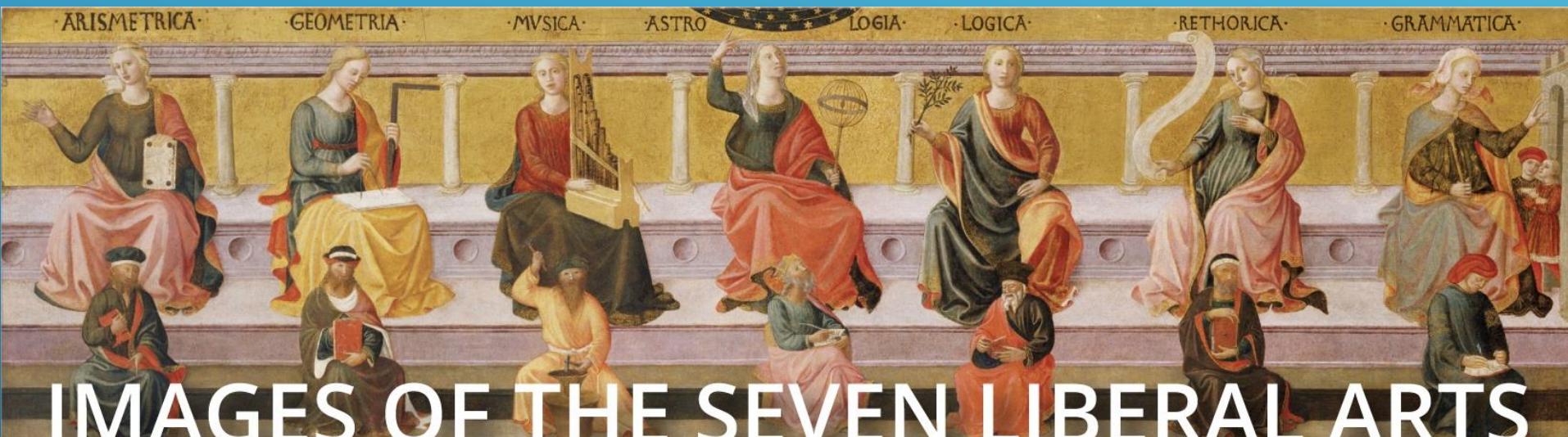
SEE: Société de l'électricité, de l'électronique et des technologies de l'information et de la communication

<https://www.see.asso.fr/>

ISCT: <https://isct7.webnode.com/>

# THE SEVEN LIBERAL ARTS: GEOMETRIA

**Liberal arts**, college or university curriculum aimed at imparting general knowledge and developing **general intellectual capacities in contrast to a professional, vocational, or technical curriculum**. In the medieval European university the seven liberal arts were **grammar, rhetoric, and logic (the trivium)** and **geometry, arithmetic, music, and astronomy (the quadrivium)**.

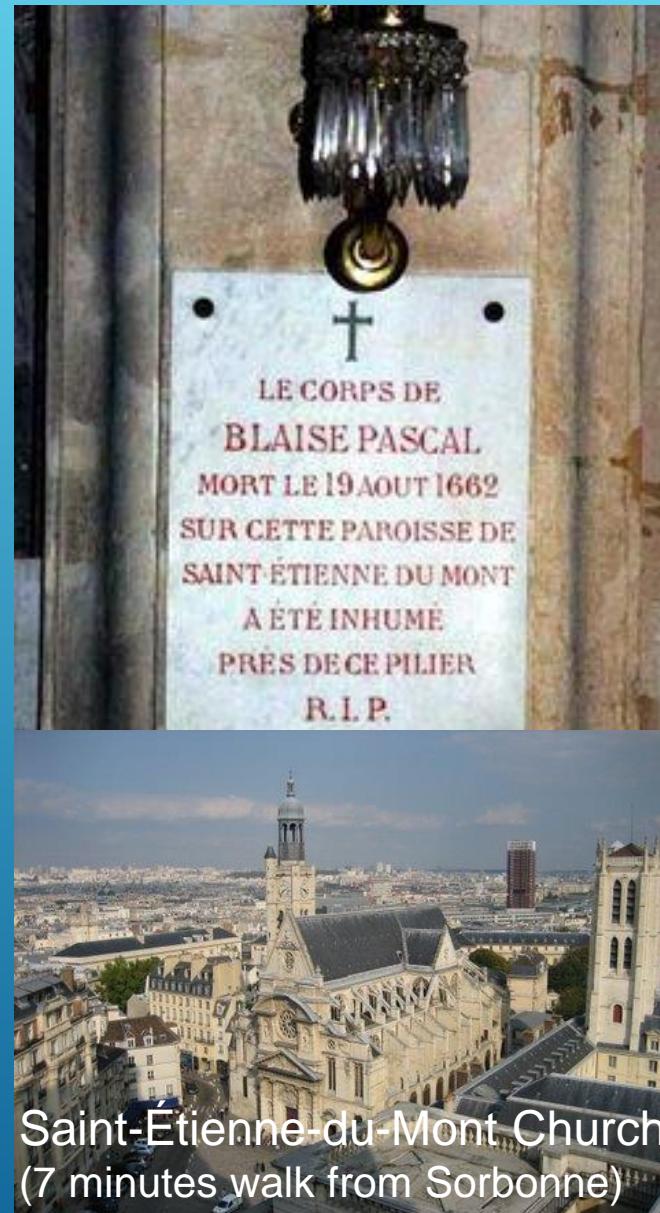


## IMAGES OF THE SEVEN LIBERAL ARTS



# BLAISE PASCAL ALEA GEOMETRIA

- ▶ In 1654, Blaise Pascal submitted a paper to « *Celeberrimae matheseos Academiae Parisiensi* » entitled « **ALEAE GEOMETRIA : De compositione aleae in ludis ipsi subjectis** »
- ▶ « ... et sic matheseos demonstrationes cum aleae incertitudine jugendo, et quae contraria videntur conciliando, ab utraque nominationem suam accipiens, stupendum hunc titulum jure sibi arrogat: **Aleae Geometria** »
- ▶ « ... par l'union ainsi réalisée entre les démonstrations des mathématiques et l'incertitude du hasard, et par la conciliation entre les contraires apparents, elle peut tirer son nom de part et d'autre et s'arroger à bon droit ce titre étonnant: **Géométrie du Hasard** »
- ▶ « ... by the union thus achieved between the demonstrations of mathematics and the uncertainty of chance, and by the conciliation between apparent opposites, it can take its name from both sides and arrogate to right this amazing title: **Geometry of Chance** »



## Blaise Pascal and DUALITY

Pascal's **Hexagrammum Mysticum Theorem**, and its dual **Brianchon's Theorem**.

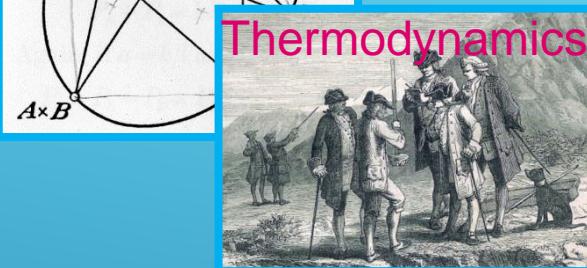
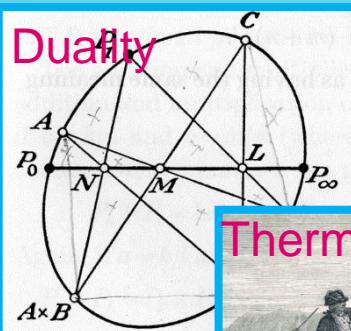
In 1639 Blaise Pascal discovered, at age sixteen, the famous hexagon theorem, also developed in "Essay pour les Coniques", printed in 1640, declaring his intention of writing a treatise on conics in which he would derive the major theorems of Apollonius from his new theorem.



Pascaline

## Blaise Pascal and COMPUTER

**Pascaline** marks the beginning of the development of mechanical calculus in Europe, followed by Charles Babbage analytical machine from 1834 to 1837, a programmable calculating machine combining the inventions of Blaise Pascal and Jacquard's machine, with instructions written on perforated cards.



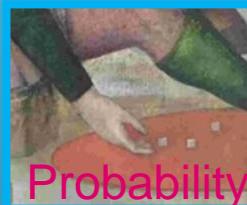
Thermodynamics



Blaise Pascal

## Blaise Pascal and THERMODYNAMICS

Pascal's Experiment in the Puy de Dôme to Test the **Relation between Atmospheric Pressure and Altitude**. In 1647, Blaise Pascal suggests to raise Torricelli's mercury barometer at the top of the Puy de Dome Mountain (France) in order to test the "weight of air" assumption



Probability

## Blaise Pascal and PROBABILITY

The "calculation of probabilities" began in a correspondence between Blaise Pascal and Pierre Fermat. In 1654, Blaise Pascal submitted a short paper to "Celeberrimae matheseos Academiae Parisiensi" with the title "**Aleae Geometria**" (**Geometry of Chance**), that was the seminal paper founding Probability as a new discipline in Science.

# Painting@La Sorbonne

Desargues, Mersenne, Pascal and Descartes



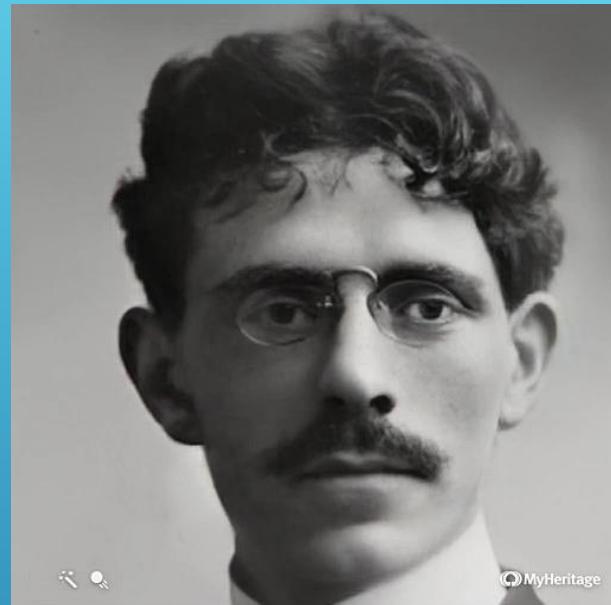
# FRENCH CONTRIBUTORS TO GEOMETRIC SCIENCE OF INFORMATION



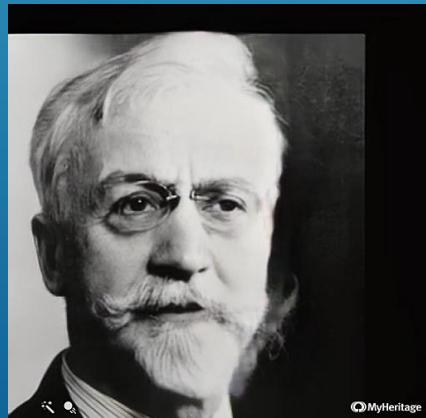
Jean-Marie Souriau (ENS 1942)



Jean-Louis Koszul (ENS 1940)



Maurice Fréchet (ENS 1903)



Elie Cartan



Henri Poincaré

# FRENCH CONTRIBUTORS TO CALCULUS OF VARIATIONS

Pierre  
de Fermat



Fermat's principle  
of least time

Pierre  
Louis  
Maupertuis



Maupertuis's  
principle of  
least length

Joseph  
Louis  
Lagrange



(Euler)  
Lagrange  
Equation

Simeon  
Denis  
Poisson



Poisson  
Bracket,  
Poisson Geometry  
Structure

Henri  
Poincaré



(Euler)  
Poincaré  
Equation

Elie  
Cartan



Poincaré  
Cartan  
Integral  
Invariant

Jean- Marie  
Souriau



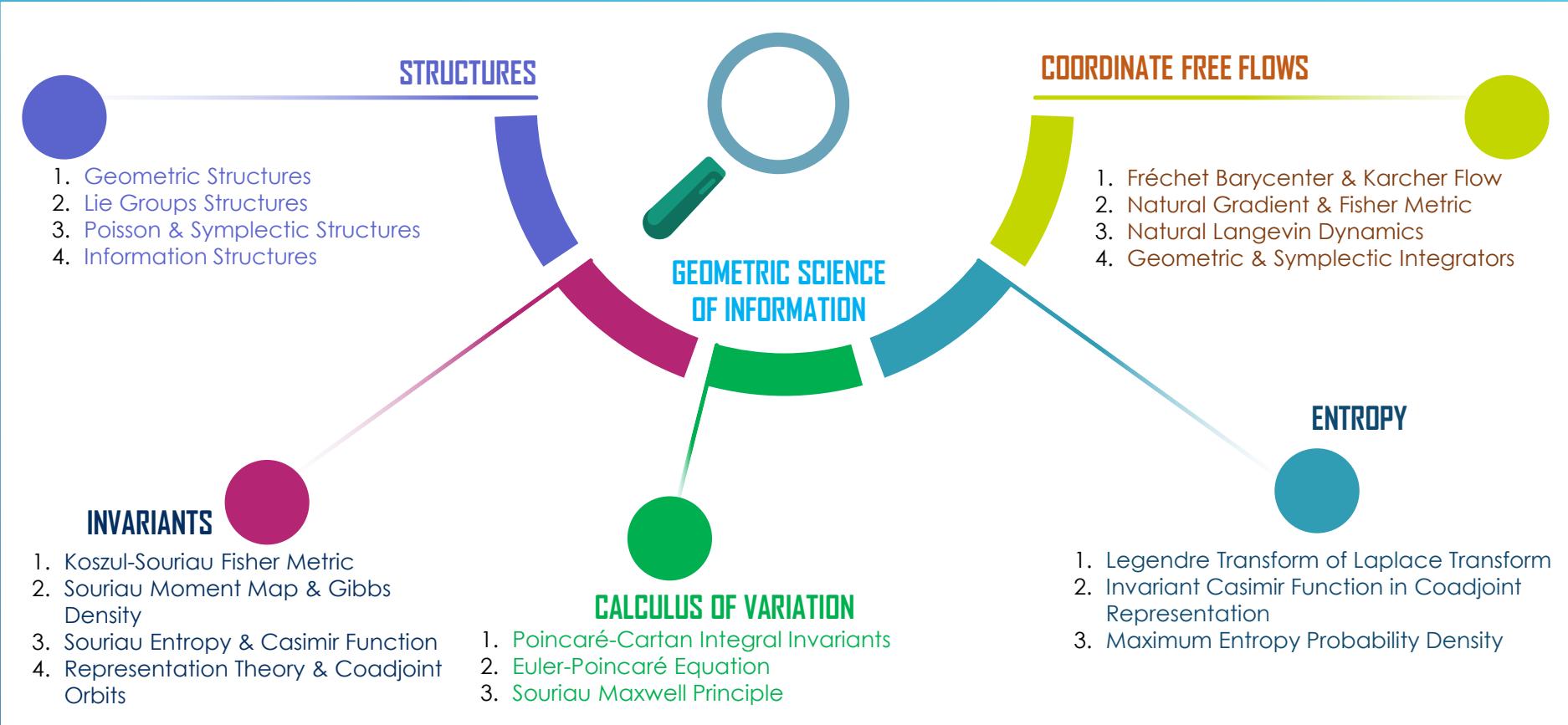
Souriau  
Moment  
Map,  
Souriau  
Symplectic  
2 Form,  
Lie Groups  
Thermodynamics

Jean-Michel  
Bismut



Random  
Mechanics

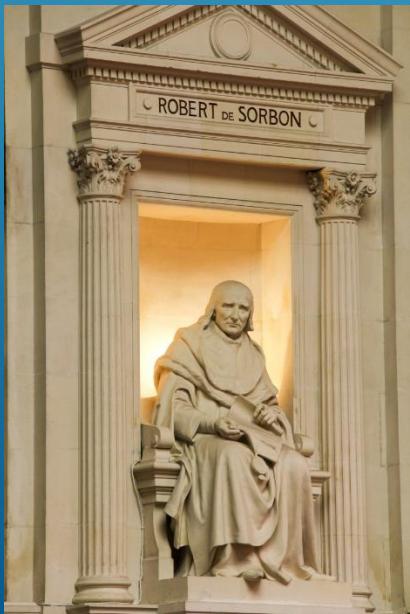
# MAIN CONCEPTS BEHIND GEOMETRIC SCIENCE OF INFORMATION

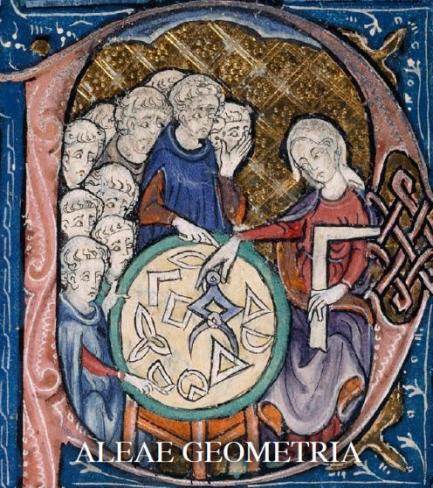


# ENJOY GSI'21



*« There is nothing more in physical theories than symmetry groups except the mathematical construction which allows precisely to show that there is nothing more » - Jean-Marie Souriau*





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**Giuseppe Longo**

Centre Cavailles, CNRS & Ens Paris and  
School of Medicine, Tufts University, Boston  
<http://www.di.ens.fr/users/longo/>

GSI'21



**5<sup>th</sup> GSI'21 Conference**

## **LEARNING GEOMETRIC STRUCTURES**

**Sorbonne University**

July 21<sup>st</sup>–22<sup>nd</sup>– 23<sup>rd</sup> 2021

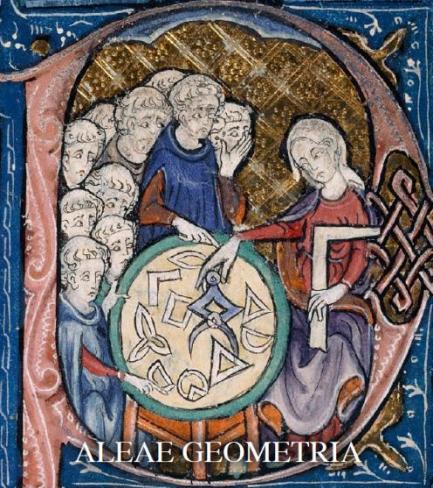
### **TITLE: USE AND ABUSE OF «DIGITAL INFORMATION» IN LIFE SCIENCES, IS GEOMETRY OF INFORMATION A WAY OUT?**

#### **Abstract:**

Since WWII, the war of coding, and the understanding of the structure of the DNA (1953), the latter has been considered as the digital encoding of the Aristotelian Homunculus. Till now DNA is viewed as the «information carrier» of ontogenesis, the main or unique player and pilot of phylogenesis. This heavily affected our understanding of life and reinforced a mechanistic view of organisms and ecosystems, a component of our disruptive attitude towards ecosystemic dynamics. A different insight into DNA as a major constraint to morphogenetic processes brings in a possible «geometry of information» for biology, yet to be invented. One of the challenges is in the need to move from a classical analysis of morphogenesis, in physical terms, to a «heterogenesis» more proper to the historicity of biology.

#### **References**

- | Arezoo Islami, Giuseppe Longo. *Marriages of Mathematics and Physics: a challenge for Biology*, Invited Paper, in The Necessary Western Conjunction to the Eastern Philosophy of Exploring the Nature of Mind and Life (K. Matsuno et al., eds), Special Issue of Progress in Biophysics and Molecular Biology, Vol 131, Pages 179–192, December 2017. (DOI) (SpaceTimelslamiLongo.pdf)
- | Giuseppe Longo. *How Future Depends on Past Histories and Rare Events in Systems of Life*, Foundations of Science, (DOI), 2017 (biolog-observ-history-future.pdf)
- | Giuseppe Longo. *Information and Causality: Mathematical Reflections on Cancer Biology*. In Organisms. Journal of Biological Sciences, vo. 2, n. 1, 2018. (BiologicalConseq-ofCompute.pdf)
- | Giuseppe Longo. *Information at the Threshold of Interpretation, Science as Human Construction of Sense*. In Bertolaso, M., Sterpetti, F. (Eds.) A Critical Reflection on Automated Science – Will Science Remain Human? Springer, Dordrecht, 2019 (Information-Interpretation.pdf)
- | Giuseppe Longo, Matteo Mossio. *Geocentrism vs genocentrism: theories without metaphors, metaphors without theories*. In Interdisciplinary Science Reviews, 45 (3), pp. 380-405, 2020. (Metaphors-geo-genocentrism.pdf)



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**Yvette Kosmann-Schwarzbach**

Professeur des universités honoraire

## TITLE: STRUCTURES OF POISSON GEOMETRY: OLD AND NEW

### Abstract:

How did the brackets that Siméon-Denis Poisson introduce in 1809 evolve into the Poisson geometry of the 1970's? What are Poisson groups and, more generally, Poisson groupoids? In what sense does Dirac geometry generalize Poisson geometry and why is it relevant for applications? I shall sketch the definition of these structures and try to answer these questions.

### References:

- | P. Libermann and C.-M. Marle, *Symplectic Geometry and Analytical Mechanics*, D. Reidel Publishing Company (1987).
- | J. E. Marsden and T. S. Ratiu, *Introduction to Mechanics and Symmetry, Texts in Applied Mathematics 17*, second edition, Springer (1998).
- | C. Laurent-Gengoux, A. Pichereau, and P. Vanhaecke, *Poisson Structures, Grundlehren der mathematischen Wissenschaften 347*, Springer (2013).
- | Y. Kosmann-Schwarzbach, *Multiplicativity from Lie groups to generalized geometry, in Geometry of Jets and Fields* (K. Grabowska et al., eds), Banach Center Publications 110, 2016.
- | Special volume of *LMP* on Poisson Geometry, guest editors, Anton Alekseev, Alberto Cattaneo, Y. Kosmann-Schwarzbach, and Tudor Ratiu, *Letters in Mathematical Physics* 90, 2009.
- | Y. Kosmann-Schwarzbach (éd.), *Siméon-Denis Poisson : les Mathématiques au service de la science*, Editions de l'Ecole Polytechnique (2013).
- | Y. Kosmann-Schwarzbach, *The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century*, translated by B. E. Schwarzbach, Sources and Studies in the History of Mathematics and Physical Sciences, Springer (2011).

GSI'21

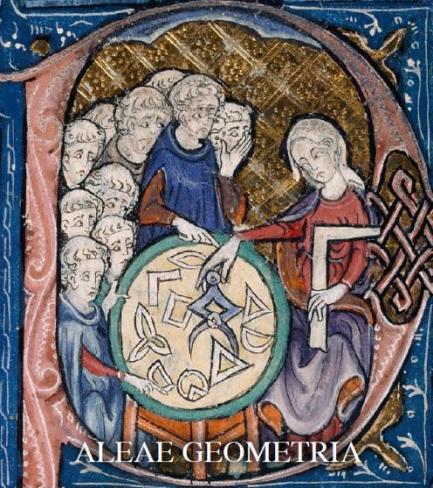


5<sup>th</sup> GSI'21 Conference

## LEARNING GEOMETRIC STRUCTURES

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**Michel Broniatowski**

Sorbonne Université, Paris

## TITLE: SOME INSIGHTS ON STATISTICAL DIVERGENCES AND CHOICE OF MODELS

### Abstract:

Divergences between probability laws or more generally between measures define inferential criteria, or risk functions. Their estimation makes it possible to deal with the questions of model choice and statistical inference, in connection with the regularity of the models considered; depending on the nature of these models (parametric or semi-parametric), the nature of the criteria and their estimation methods vary. Representations of these divergences as large deviation rates for specific empirical measures allow their estimation in nonparametric or semi parametric models, by making use of information theory results (Sanov's theorem and Gibbs principles), by Monte Carlo methods. The question of the choice of divergence is wide open; an approach linking nonparametric Bayesian statistics and MAP estimators provides elements of understanding of the specificities of the various divergences in the Ali-Silvey-Csiszar-Arimoto class in relation to the specific choices of the prior distributions.

### References:

- Broniatowski, Michel ; Stummer, Wolfgang. *Some universal insights on divergences for statistics, machine learning and artificial intelligence*. In Geometric structures of information; Signals Commun. Technol., Springer, Cham, pp. 149-211, 2019
- Broniatowski, Michel. *Minimum divergence estimators, Maximum Likelihood and the generalized bootstrap*, to appear in «Divergence Measures: Mathematical Foundations and Applications in Information-Theoretic and Statistical Problems» Entropy, 2020
- Csiszár, Imre ; Gassiat, Elisabeth. *MEM pixel correlated solutions for generalized moment and interpolation problems*. IEEE Trans. Inform. Theory 45, no. 7, 2253–2270, 1999
- Liese, Friedrich; Vajda, Igor. *On divergences and informations in statistics and information theory*. IEEE Trans. Inform. Theory 52, no. 10, 4394–4412, 2006

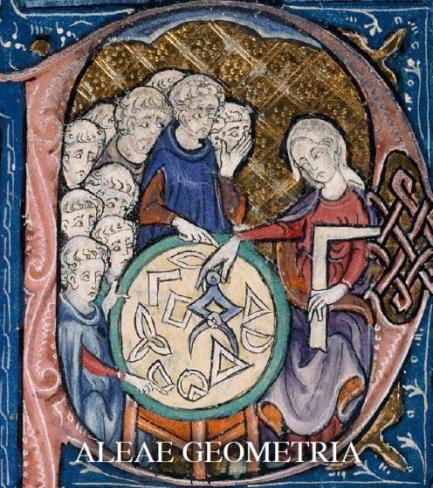


5<sup>th</sup> GSI'21 Conference

## LEARNING GEOMETRIC STRUCTURES

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**Maurice de Gosson**

Professor, Senior Researcher at the University of Vienna <https://homepage.univie.ac.at/maurice.de.gosson>  
Faculty of Mathematics, NuHAG group



**5<sup>th</sup> GSI'21 Conference  
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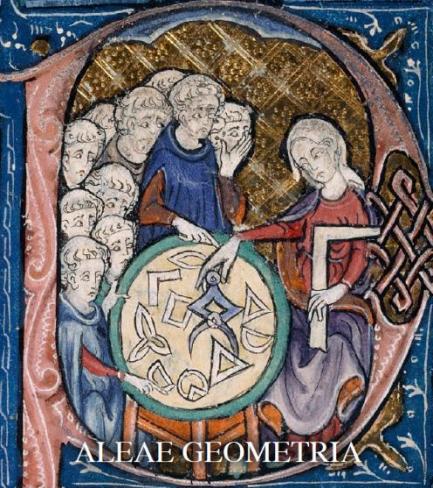
## TITLE: GAUSSIAN STATES FROM A SYMPLECTIC GEOMETRY POINT OF VIEW

### Abstract:

Gaussian states play an ubiquitous role in quantum information theory and in quantum optics because they are easy to manufacture in the laboratory, and have in addition important extremality properties. Of particular interest are their separability properties. Even if major advances have been made in their study in recent years, the topic is still largely open. In this talk we will discuss separability questions for Gaussian states from a rigorous point of view using symplectic geometry, and present some new results and properties.

### References:

- | Broniatowski, Michel ; Stummer, Wolfgang. *Some universal insights on divergences for statistics, machine learning and artificial intelligence*. In Geometric structures of information; Signals Commun. Technol., Springer, Cham, pp. 149.211, 2019
- | Broniatowski, Michel. *Minimum divergence estimators, Maximum Likelihood and the generalized bootstrap*, to appear in «Divergence Measures: Mathematical Foundations and Applications in Information-Theoretic and Statistical Problems» Entropy, 2020
- | Csiszár, Imre ; Gassiat, Elisabeth. *MEM pixel correlated solutions for generalized moment and interpolation problems*. IEEE Trans. Inform. Theory 45, no. 7, 2253–2270, 1999
- | Liese, Friedrich; Vajda, Igor. *On divergences and informations in statistics and information theory*. IEEE Trans. Inform. Theory 52, no. 10, 4394–4412, 2006



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see



**Max Welling**

Informatics Institute, University of Amsterdam and Qualcomm Technologies  
<https://staff.fnwi.uva.nl/m.welling>  
ELLIS Board Member (European Laboratory for Learning and Intelligent Systems):  
<https://ellis.eu>

## TITLE: EXPLORING QUANTUM STATISTICS FOR MACHINE LEARNING

### Abstract:

Quantum mechanics represents a rather bizarre theory of statistics that is very different from the ordinary classical statistics that we are used to. In this talk I will explore if there are ways that we can leverage this theory in developing new machine learning tools: can we design better neural networks by thinking about entangled variables?

Can we come up with better samplers by viewing them as observations in a quantum system? Can we generalize probability distributions? We hope to develop better algorithms that can be simulated efficiently on classical computers, but we will naturally also consider the possibility of much faster implementations on future quantum computers. Finally, I hope to discuss the role of symmetries in quantum theories.

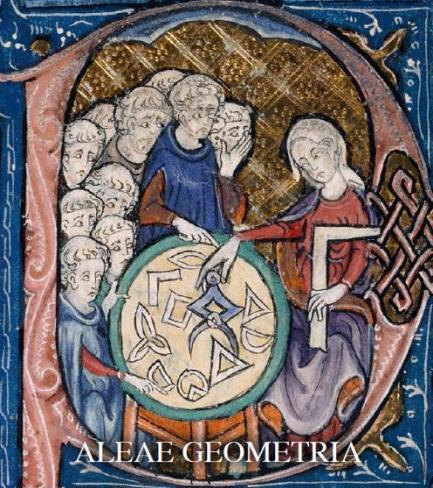
### References:

- Roberto Bondesan, Max Welling, *Quantum Deformed Neural Networks*, arXiv:2010.11189v1 [quant-ph], 21<sup>st</sup> October 2020 ; <https://arxiv.org/abs/2010.11189>



## 5<sup>th</sup> GSI'21 Conference LEARNING GEOMETRIC STRUCTURES

Sorbonne University  
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**Jean Petitot**

Directeur d'Études, Centre d'Analyse et de Mathématiques, Sociales, École des Hautes Études, Paris.

## TITLE : THE PRIMARY VISUAL CORTEX AS A CARTAN ENGINE

### Abstract:

Cortical visual neurons detect very local geometric cues as retinal positions, local contrasts, local orientations of boundaries, etc. One of the main theoretical problem of low level vision is to understand how these local cues can be integrated so as to generate the global geometry of the images perceived, with all the well-known phenomena studied since Gestalt

theory. It is an empirical evidence that the visual brain is able to perform a lot of routines belonging to differential geometry. But how such routines can be neurally implemented ? Neurons are « point-like » processors and it seems impossible to do differential geometry with them. Since the 1990s, methods of «*in vivo* optical imaging based on activity-dependent intrinsic signals» have made possible to visualize the extremely special connectivity of the primary visual areas, their “functional architectures.” What we called «Neurogeometry» is based on the discovery that these functional architectures implement structures such as the contact structure and the sub-Riemannian geometry of jet spaces of plane curves. For reasons of principle, it is the geometrical reformulation of differential calculus from Pfaff to Lie, Darboux, Frobenius, Cartan and Goursat which turns out to be suitable for neurogeometry.

### References:

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## LEARNING GEOMETRIC STRUCTURES

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