

**5<sup>th</sup> Conference on  
the Geometric Science  
of Information**

**GSI'21**

**Sorbonne University**

**July 21<sup>st</sup> – 22<sup>nd</sup> – 23<sup>rd</sup> 2021**



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unit

PARIS



SORBONNE CENTER FOR  
ARTIFICIAL INTELLIGENCE





# 5<sup>th</sup> GSI'21 Conference

## LEARNING GEOMETRIC STRUCTURES

**Sorbonne University**

**July 21<sup>st</sup> – 22<sup>nd</sup> – 23<sup>rd</sup> 2021**

**A**s for GSI'13, GSI'15, GSI'17 and GSI'19 (<https://franknielsen.github.io/GSI/>), the objective of this 5th edition of SEE GSI'21 conference, hosted in PARIS, is to bring together pure/applied mathematicians and engineers, with common interest for Geometric tools and their applications for Information analysis. It emphasizes an active participation of young researchers to discuss emerging areas of collaborative research on “Geometric Science of Information and their Applications”. In 2021, GSI main topics was on “LEARNING GEOMETRIC STRUCTURES” and the conference took place at Sorbonne University.

The GSI conference cycle has been initiated by the Brillouin Seminar Team as soon as 2009 (<http://repmus.ircam.fr/brillouin/home>). The GSI'21 event has been motivated in the continuity of first initiatives launched in 2013 (<https://www.see.asso.fr/gsi2013>) at Mines ParisTech, consolidated in 2015 (<https://www.see.asso.fr/gsi2015>) at Ecole Polytechnique and opened to new communities in 2017 (<https://www.see.asso.fr/gsi2017>) at Mines ParisTech and 2019 (<https://www.see.asso.fr/gsi2019>) at ENAC Toulouse. We mention that in 2011, we organized an indo-french workshop on “Matrix Information Geometry” that yielded an edited book in 2013, and in 2017, collaborate to

CIRM seminar in Luminy TGSi'17 "Topological & Geometrical Structures of Information" (<http://forum.cs-dc.org/category/94/tgsi2017>).

GSI satellites event have been organized in 2019 and 2020 as, FGSI'19 "Foundation of Geometric Science of Information" in Montpellier (<https://fgsi2019.sciencesconf.org/>) and Les Houches Seminar SPIGL'20 "Joint Structures and Common Foundations of Statistical Physics, Information Geometry and Inference for Learning" (<https://www.springer.com/jp/book/9783030779566>).

The technical program of GSI'21 covers all the main topics and highlights in the domain of "Geometric Science of Information" including Information Geometry Manifolds of structured data/information and their advanced applications. GSI'21 Proceedings consists solely of original research papers that have been carefully peer-reviewed by two or three experts before, and revised before acceptance.

As for the GSI'13, GSI'15, GSI'17, and GSI'19, GSI'21 addresses inter-relations between different mathematical domains like shape spaces (geometric statistics on manifolds and Lie groups, deformations in shape space, ...), probability/optimization & algorithms on manifolds (structured matrix manifold, structured data/Information, ...), relational and discrete metric spaces (graph metrics, distance geometry, relational analysis,...), computational and Hessian information geometry, geometric structures in thermodynamics and statistical physics, algebraic/infinite dimensional/Banach information manifolds, divergence geometry, tensor-valued morphology, optimal transport theory, manifold & topology learning, ... and applications like geometries of audio-processing, inverse problems and signal/image processing. GSI'21 topics were enriched with contributions from Lie Group Machine Learning, Harmonic Analysis on Lie Groups, Geometric Deep Learning, Geometry of Hamiltonian Monte Carlo, Geometric & (Poly)Symplectic Integrators, Contact Geometry & Hamiltonian Control, Geometric and structure preserving discretizations, Probability Density Estimation & Sampling in High Dimension, Geometry of Graphs and Networks and Geometry in Neuroscience & Cognitive Sciences.

At the turn of the century, new and fruitful interactions were discovered between several branches of science: Information Science (information theory, digital communications, statistical signal processing,), Mathematics (group theory, geometry and topology, probability, statistics, sheaves theory,...) and Physics (geometric mechanics, thermodynamics, statistical physics, quantum mechanics, ...). GSI conference cycle is a tentative to discover joint mathematical structures to all these disciplines by elaboration of a "General Theory of Information" embracing physics science, information science, and cognitive science in a global scheme. ■



## GSI'21 conference was structured in 22 sessions:

**Probability & statistics on Riemannian Manifolds** – Chairs: Xavier Pennec, Cyrus Mostajeran

**Shapes Spaces** – Chairs: Salem Said, Joan Glaunès

**Geometric and structure preserving discretizations** –

Chairs: Alessandro Bravetti, Manuel de Leon

**Lie Group Machine Learning** – Chairs: Frédéric Barbaresco, Gery de Saxcé

**Harmonic Analysis on Lie Groups** – Chairs: Jean-Pierre Gazeau, Frédéric Barbaresco

**Geometric Mechanics** – Chairs: Gery de Saxcé, Frédéric Barbaresco

**Sub-Riemannian Geometry and Neuromathematics** – Chairs: Alessandro Sarti, Dario Prandi

**Statistical Manifold & Hessian Information Geometry** –

Chairs: Noemie Combe, Michel Nguiffo Boyom

**Information Geometry in Physics** – Chairs: Geert Verdoolaage, Jun Zhang

**Geometric & Symplectic Methods for Hydrodynamical Models** –

Chairs: Cesare Tronci, François Gay-Balmaz

**Geometry of Quantum States** – Chairs: Florio Maria Ciaglia, Michel Berthier

**Deformed entropy, cross-entropy, and relative entropy** –

Chairs: Ting-Kam Leonard Wong, Léonard Monsaingeon

**Geometric structures in thermodynamics and statistical physics** –

Chairs: Hiroaki Yoshimura, François Gay-Balmaz

**Geometric Deep Learning** – Chairs: Gabriel Peyré, Erik J. Bekkers

**Computational Information Geometry 1** – Chairs: Frank Nielsen, Clément Gauchy

**Computational Information Geometry 2** – Chairs: Giovanni Pistone, Goffredo Chirco

**Optimal Transport & Learning** – Chairs: Yaël Frégier, Nicolas Garcia Trillos

**Statistics, Information and Topology** – Chairs: Pierre Baudot, Michel Nguiffo Boyom

**Topological and Geometrical Structures in Neurosciences** – Chairs: Pierre Baudot, Giovanni Petri

**Manifolds & Optimization** – Chairs: Stéphanie Jehan-Besson, Bin Gao

**Divergence Statistics** – Chairs: Michel Broniatowski, Wolfgang Stummer

**Transport information geometry** – Chairs: Wuchen Li, Philippe Jacquet

## | GSI 2021 General Chairs



**Frédéric Barbaresco**  
THALES



**Frank Nielsen**  
Sony CSL

# I Keynote Speakers

## Michel Broniatowski

Sorbonne Université, Paris



### TITLE: SOME INSIGHTS ON STATISTICAL DIVERGENCES AND CHOICE OF MODELS

#### Abstract:

Divergences between probability laws or more generally between measures define inferential criteria, or risk functions. Their estimation makes it possible to deal with the questions of model choice and statistical inference, in connection with the regularity of the models considered; depending on the nature of these models (parametric or semi-parametric), the nature of the criteria and their estimation methods vary. Representations of these divergences as large deviation rates for specific empirical measures allow their estimation in nonparametric or semi parametric models, by making use of information theory results (Sanov's theorem and Gibbs principles), by Monte Carlo methods. The question of the choice of divergence is wide open; an approach linking nonparametric Bayesian statistics and MAP estimators provides elements of understanding of the specificities of the various divergences in the Ali-Silvey-Csiszar-Arimoto class in relation to the specific choices of the prior distributions.

#### References:

- I Broniatowski, Michel ; Stummer, Wolfgang. **Some universal insights on divergences for statistics, machine learning and artificial intelligence**. In Geometric structures of information; Signals Commun. Technol., Springer, Cham, pp. 149.211, 2019
- I Broniatowski, Michel. **Minimum divergence estimators, Maximum Likelihood and**

*the generalized bootstrap*, to appear in «Divergence Measures: Mathematical Foundations and Applications in Information-Theoretic and Statistical Problems» Entropy, 2020

- I Csiszár, Imre ; Gassiat, Elisabeth. **MEM pixel correlated solutions for generalized moment and interpolation problems**. IEEE Trans. Inform. Theory 45, no. 7, 2253–2270, 1999
- I Liese, Friedrich; Vajda, Igor. **On divergences and informations in statistics and information theory**. IEEE Trans. Inform. Theory 52, no. 10, 4394–4412, 2006

## Maurice de Gosson

Professor, Senior Researcher at the University of Vienna <https://homepage.univie.ac.at/maurice.de.gosson>  
Faculty of Mathematics, NuHAG group



### TITLE: GAUSSIAN STATES FROM A SYMPLECTIC GEOMETRY POINT OF VIEW

#### Abstract:

Gaussian states play an ubiquitous role in quantum information theory and in quantum optics because they are easy to manufacture in the laboratory, and have in addition important extremality properties. Of particular interest are their separability properties. Even if major advances have been made in their study in recent years, the topic is still largely open. In this talk we will discuss separability questions for Gaussian states from a rigorous point of view using symplectic geometry, and present some new results and properties.

**References:**

| M. de Gosson, On the Disentanglement of Gaussian Quantum States by Symplectic Rotations. C.R. Acad. Sci. Paris Volume 358, issue 4, 459-462 (2020)

| M. de Gosson, On Density Operators with Gaussian Weyl symbols, In Advances in Microlocal and Time-Frequency Analysis, Springer (2020)

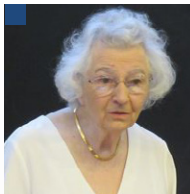
| M. de Gosson, Symplectic Coarse-Grained Classical and Semiclassical Evolution of Subsystems: New Theoretical Aspects, J. Math. Phys. no. 9, 092102 (2020)

| E. Cordero, M. de Gosson, and F. Nicola, On the Positivity of Trace Class Operators, to appear in Advances in Theoretical and Mathematical Physics 23(8), 2061–2091 (2019)

| E. Cordero, M. de Gosson, and F. Nicola, A characterization of modulation spaces by symplectic rotations, to appear in J. Funct. Anal. 278(11), 108474 (2020)

**Yvette Kosmann-Schwarzbach**

*Professeur des universités honoraire ; former student of the Ecole normale supérieure Sèvres, 1960-1964; aggregation of mathematics, 1963; CNRS research associate, 1964-1969; doctorate in science, Lie derivatives of spinors, University of Paris, 1970 under supervision of André Lichnerowicz; lecturer then professor at the University of Lille (1970-1976 and 1982-1993), at Brooklyn College, New York (1979-1982), at the École polytechnique (1993-2006)*



**TITLE: STRUCTURES OF POISSON GEOMETRY: OLD AND NEW**

**Abstract:**

How did the brackets that Siméon-Denis Poisson introduce in 1809 evolve into the Poisson geometry of the 1970's? What are Poisson groups and, more generally, Poisson groupoids? In what sense does Dirac geometry generalize Poisson geometry and why is it relevant for applications? I shall sketch the definition of these structures and try to answer these questions.

**References:**

| P. Libermann and C.-M. Marle, *Symplectic Geometry and Analytical Mechanics*, D. Reidel Publishing Company (1987).

| J. E. Marsden and T. S. Ratiu, *Introduction to Mechanics and Symmetry, Texts in Applied Mathematics 17*, second edition, Springer (1998).

| C. Laurent-Gengoux, A. Pichereau, and P. Vanhaecke, *Poisson Structures, Grundlehren der mathematischen Wissenschaften 347*, Springer (2013).

| Y. Kosmann-Schwarzbach, *Multiplicativity from Lie groups to generalized geometry, in Geometry of Jets and Fields* (K. Grabowska et al., eds), Banach Center Publications 110, 2016.

| *Special volume of LMP on Poisson Geometry*, guest editors, Anton Alekseev, Alberto Cattaneo, Y. Kosmann-Schwarzbach, and Tudor Ratiu, *Letters in Mathematical Physics 90*, 2009.

| Y. Kosmann-Schwarzbach (éd.), Siméon-Denis Poisson : *les Mathématiques au service de la science, Editions de l'Ecole Polytechnique* (2013).

| Y. Kosmann-Schwarzbach, *The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century*, translated by B. E. Schwarzbach, Sources and Studies in the History of Mathematics and Physical Sciences, Springer (2011).

**Giuseppe Longo**

Centre Cavailles, CNRS & Ens Paris and School of Medicine, Tufts University, Boston <http://www.di.ens.fr/users/longo/>



## TITLE: USE AND ABUSE OF «DIGITAL INFORMATION» IN LIFE SCIENCES, IS GEOMETRY OF INFORMATION A WAY OUT?

### Abstract:

Since WWII, the war of coding, and the understanding of the structure of the DNA (1953), the latter has been considered as the digital encoding of the Aristotelian Homunculus. Till now DNA is viewed as the «information carrier» of ontogenesis, the main or unique player and pilot of phylogenesis. This heavily affected our understanding of life and reinforced a mechanistic view of organisms and ecosystems, a component of our disruptive attitude towards ecosystemic dynamics. A different insight into DNA as a major constraint to morphogenetic processes brings in a possible «geometry of information» for biology, yet to be invented. One of the challenges is in the need to move from a classical analysis of morphogenesis, in physical terms, to a «heterogenesis» more proper to the historicity of biology.

### References

- I Arezoo Islami, Giuseppe Longo. *Marriages of Mathematics and Physics: a challenge for Biology*, Invited Paper, in The Necessary Western Conjunction to the Eastern Philosophy of Exploring the Nature of Mind and Life (K. Matsuno et al., eds), Special Issue of Progress in Biophysics and Molecular Biology, Vol 131, Pages 179–192, December 2017. (DOI) (SpaceTimeIslamiLongo.pdf)
- I Giuseppe Longo. *How Future Depends on Past Histories and Rare Events in Systems of Life*, Foundations of Science, (DOI), 2017 (biolog-observ-history-future.pdf)
- I Giuseppe Longo. *Information and Causality: Mathematical Reflections on Cancer Biology*. In Organisms. Journal of Biological Sciences, vo. 2, n. 1, 2018. (BiologicalConseq-ofCompute.pdf)

I Giuseppe Longo. *Information at the Threshold of Interpretation, Science as Human Construction of Sense*. In Bertolaso, M., Sterpetti, F. (Eds.) A Critical Reflection on Automated Science – Will Science Remain Human? Springer, Dordrecht, 2019 (Information-Interpretation.pdf)

I Giuseppe Longo, Matteo Mossio. *Geocentrism vs genocentrism: theories without metaphors, metaphors without theories*. In Interdisciplinary Science Reviews, 45 (3), pp. 380-405, 2020. (Metaphors-geo-genocentrism.pdf)

## Jean Petitot

Directeur d'Études, Centre d'Analyse et de Mathématiques, Sociales, École des Hautes Études, Paris.

Born in 1944, Jean Petitot is an applied mathematician interested in dynamical modeling in neurocognitive sciences. He is the former director of the CREA (Applied Epistemology Research Center) at the Ecole Polytechnique. Philosopher of science <http://jeanpetitot.com>



## TITLE : THE PRIMARY VISUAL CORTEX AS A CARTAN ENGINE

### Abstract:

Cortical visual neurons detect very local geometric cues as retinal positions, local contrasts, local orientations of boundaries, etc. One of the main theoretical problem of low level vision is to understand how these local cues can be integrated so as to generate the global geometry of the images perceived, with all the well-known phenomena studied since Gestalt

theory. It is an empirical evidence that the visual brain is able to perform a lot of routines belonging to differential geometry. But how such routines can be neurally implemented ? Neurons are « point-like » processors and it seems impossible to do differential geometry with them. Since the 1990s, methods of «in vivo optical imaging based on activity-dependent intrinsic signals» have made possible to visualize the extremely special connectivity of the primary visual areas, their “functional architectures.” What we called «Neurogeometry » is based on the discovery that these functional architectures implement structures such as the contact structure and the sub-Riemannian geometry of jet spaces of plane curves. For reasons of principle, it is the geometrical reformulation of differential calculus from Pfaff to Lie, Darboux, Frobenius, Cartan and Goursat which turns out to be suitable for neurogeometry.

**References:**

- ┃ Agrachev, A., Barilari, D., Boscain, U., *A Comprehensive Introduction to Sub-Riemannian Geometry*, Cambridge University Press, 2020.
- ┃ Citti, G., Sarti, A., *A cortical based model of perceptual completion in the roto-translation space*, Journal of Mathematical Imaging and Vision, 24, 3 (2006) 307-326.
- ┃ Petitot, J., *Neurogéométrie de la vision. Modèles mathématiques et physiques des architectures fonctionnelles*, Les Éditions de l'École Polytechnique, Distribution Ellipses, Paris, 2008.
- ┃ Petitot, J., “*Landmarks for neurogeometry*”, *Neuromathematics of Vision*, (G. Citti, A. Sarti eds), Springer, Berlin, Heidelberg, 1-85, 2014.
- ┃ Petitot, J., *Elements of Neurogeometry. Functional Architectures of Vision, Lecture Notes in Morphogenesis*, Springer, 2017.
- ┃ Prandi, D., Gauthier, J.-P., *A Semidiscrete Version of the Petitot Model as a Plausible Model for Anthropomorphic Image Reconstruction and Pattern Recognition*, <https://arxiv.org/abs/1704.03069v1>, 2017.

**Max Welling**

Informatics Institute, University of Amsterdam and Qualcomm Technologies  
<https://staff.fnwi.uva.nl/m.welling>  
 ELLIS Board Member (European Laboratory for Learning and Intelligent Systems: <https://ellis.eu>)



**TITLE: EXPLORING QUANTUM STATISTICS FOR MACHINE LEARNING**

**Abstract:**

Quantum mechanics represents a rather bizarre theory of statistics that is very different from the ordinary classical statistics that we are used to. In this talk I will explore if there are ways that we can leverage this theory in developing new machine learning tools: can we design better neural networks by thinking about entangled variables? Can we come up with better samplers by viewing them as observations in a quantum system? Can we generalize probability distributions? We hope to develop better algorithms that can be simulated efficiently on classical computers, but we will naturally also consider the possibility of much faster implementations on future quantum computers. Finally, I hope to discuss the role of symmetries in quantum theories.

**References:**

- ┃ Roberto Boudesan, Max Welling, *Quantum Deformed Neural Networks*, arXiv:2010.11189v1 [quant-ph], 21<sup>st</sup> October 2020 ; <https://arxiv.org/abs/2010.11189>



# Organization of the Sessions

## PLENARY SESSION JULY 21<sup>ST</sup>

GSI'21 OPENING DAY, July 21 <sup>st</sup> Amphi RICHELIEU - Sorbonne Université		
08.30-09.00	Welcome Desk	
09.00-09.30	<b>Opening by GSI'21 chairs &amp; sponsors</b> Frédéric BARBARESCO & Frank NIELSEN	
09.30-10.30	<b>Use and abuse of «digital information» in life sciences, is Geometry of Information a way out?</b> Giuseppe LONGO - Centre Cavallès, CNRS & Ens Paris and School of Medicine, Tufts University, Boston	Keynote chair: Frédéric BARBARESCO
10.30-11.00	<b>Coffee Break + Group Photo</b>	
11.00-12.00	<b>Structures of Poisson geometry: old and new</b> Yvette KOSMANN-SCHWARZBACH - Professeur des universités honoraire - France	Keynote chair: Gery de SAXCE
12.00-14.00	<b>Lunch Break</b> ( <i>not included</i> ) + mini-hackathon GEOMSTATS at SCAI	Mini-hackathon: Nina Miolane
14.00-15.00	<b>Some insights on statistical divergences and choice of models</b> Michel BRONIATOWSKI - Sorbonne Université, France	Keynote Chair: Frank NIELSEN
15.00-16.00	<b>Gaussian states from a symplectic geometry point of view</b> Maurice de GOSSON - Faculty of Mathematics, NuHAG group - University of Vienna, Austria	Keynote Chair: Jean-Pierre GAZEAU
16.00-16.30	<b>Coffee Break</b>	
16.30-17.30	<b>Exploring quantum statistics for machine learning</b> Max WELLING, Informatics Institute, University of Amsterdam and Qualcomm Technologies	Keynote Chair: Gabriel PEYRE
17.30-18.30	<b>The primary visual cortex as a Cartan engine</b> Jean PETITOT - CAMS, EHESS (Ecole des Hautes Etudes en Sciences Sociales)	Keynote Chair: Daniel BENNEQUIN
20.00-21.30	<b>Welcome cocktail</b> - SCAI Sorbonne (Jussieu site)	

SESSIONS JULY 22<sup>ND</sup>

<b>GSI'21, July 22<sup>nd</sup></b> <b>Jussieu site - Sorbonne Université</b>		
	SESSIONS ROOM A	SESSIONS ROOM B
08.30-10.10	<b>Probability &amp; statistics on Riemannian Manifolds</b> Chairs: Xavier Pennec, Cyrus Mostajeran	<b>Sub-Riemannian Geometry and Neuromathematics</b> Chairs: Alessandro Sarti, Dario Prandi
10.10-10.30	<b>Coffee Break + Posters Session 1</b>	
10.30-12.10	<b>Shapes Spaces</b> Chairs: Salem Said, Joan Glaunès	<b>Geometry of Quantum States</b> Chairs: Florio Maria Ciaglia, Michel Berthier
12.10-13.10	<b>Lunch Break</b> ( <i>not included</i> ) + mini-hackathon GEOMSTATS at SCAI	
13.10-14.30	<b>Geometric and structure preserving discretizations</b> Chairs: Alessandro Bravetti, Manuel de Leon	<b>Information Geometry in Physics</b> Chairs: Geert Verdoolaege, Jun Zhang
14.30-16.10	<b>Lie Group Machine Learning</b> Chairs: Frédéric Barbaresco, Gery de Saxcé	<b>Geometric &amp; Symplectic Methods for Hydrodynamical Models</b> Chairs: Cesare Tronci, François Gay-Balmaz
16.10-16.30	<b>Coffee Break + Posters Session 1</b>	
16.30-18.10	<b>Harmonic Analysis on Lie Groups</b> Chairs: Jean-Pierre Gazeau, Frédéric Barbaresco	<b>Statistical Manifold &amp; Hessian Information Geometry</b> Chairs: Noemie Combe, Michel Nguiffo Boyom
18.10-19.30	<b>Geometric Mechanics</b> Chairs: Gery de Saxcé, Frédéric Barbaresco	<b>Deformed entropy, cross-entropy, and relative entropy</b> Chairs: Ting-Kam Leonard Wong, Léonard Monsaingeon
20.00-22.30	<b>GALA Dinner</b>	

SESSIONS JULY 23<sup>RD</sup>

<b>GSI'21, July 23<sup>rd</sup></b> <b>Jussieu site – Sorbonne Université</b>		
	<b>SESSIONS ROOM A</b>	<b>SESSIONS ROOM B</b>
08.30–09.50	<b>Transport information geometry</b> Chairs: Wuchen Li, Philippe Jacquet	<b>Statistics, Information and Topology</b> Chairs: Pierre Baudot, Michel Nguiffo Boyom
09.50–10.10	<b>Coffee Break + Posters Session 2</b>	
10.10–11.50	<b>Geometric Deep Learning</b> Chairs: Gabriel Peyré, Erick J. Bekkers	<b>Topological and Geometrical Structures in Neurosciences</b> Chairs: Pierre Baudot, Giovanni Petri
11.50–13.00	<b>Lunch Break</b> ( <i>not included</i> ) + mini-hackathon GEOMSTATS at SCAI	
13.00–14.40	<b>Computational Information Geometry 1</b> Chairs: Frank Nielsen, Clément Gauchy	<b>Manifolds &amp; Optimization</b> Chairs: Stéphanie Jehan-Besson, Bin Gao
14.40–16.20	<b>Computational Information Geometry 2</b> Chairs: Giovanni Pistone, Goffredo Chirco	<b>Divergence Statistics</b> Chairs: Michel Broniatowski, Wolfgang Stummer
16.20–16.40	<b>Coffee Break + Posters Session 2</b>	
16.40–18.00	<b>Optimal Transport &amp; Learning</b> Chairs: Yaël Frégier, Nicolas Garcia Trillos	<b>Geometric structures in thermodynamics and statistical physics</b> Chairs: Hiroaki Yoshimura, François Gay-Balmaz
18.00–18.30	<b>GSI'21 Closure</b>	

# Event Map for GSi'21



## Sessions Locations

July 21<sup>st</sup>

Amphithéâtre Richelieu  
Paris-Sorbonne University  
17 rue de la Sorbonne, 75005 Paris

July 22<sup>nd</sup> & 23<sup>rd</sup>

Centre International  
de Conférences  
Sorbonne Université  
Patio 44, 4 place Jussieu 55,  
75005 Paris



## Welcome Cocktail

July 21<sup>st</sup>

SCAI  
Sorbonne Université  
Campus Pierre et Marie Curie 4  
Place Jussieu, 75005 Paris



## Gala Dinner Boat Cruise

July 22<sup>nd</sup>

Boarding Point Canauxrama  
Port de l'Arsenal  
Port on Canal Saint-Martin facing  
50 bd de la Bastille, 75012 Paris

## Wednesday, July 21<sup>st</sup>

### GSI'21 Opening Day

Paris Sorbonne University

Location: Amphithéâtre RICHELIEU

- 8:30-9:00**    **Welcome Desk - get your badge**
- 9:00-9:30**    **Opening by GSI'21 Chairs & Sponsors**  
Frédéric BARBARESCO & Frank NIELSEN
- 9:30-10:30**    **Keynote Speech: Use and Abuse of “digital information” in life sciences, is Geometry of Information a way out?**  
Giuseppe LONGO - Centre Cavailles, CNRS & Ens Paris and School of Medicine, Tufts University, Boston
- 10:30-11:00**    **Coffee break + Group photo**
- 11:00-12:00**    **Structures of Poisson geometry: Old and New**  
Yvette KOSMANN-SCHWARZBACH - Professeur des universités honoraire, France
- 12:00-14:00**    **Lunch Break** (*not included*)
- 14:00-15:00**    **Some Insights on Statistical Divergences and Choice of Models**  
Michel BRONIATOWSKI - Sorbonne University
- 15:00-16:00**    **Gaussian States from a Symplectic Geometry Point of View**  
Maurice de GOSSON - Faculty of Mathematics, NuHAG group - University of Vienna, Austria
- 16:00-16:30**    **Coffee Break**
- 16:30-17:30**    **Exploring Quantum Statistics for Machine Learning**  
Max WELLING - Informatics Institute, University of Amsterdam and Qualcomm Technologies
- 17:30-18:30**    **The Primary Visual Cortex as a Cartan Engine**  
Jean PETITOT - CAMS, EHESS (École des Hautes Etudes en Sciences Sociales)
- 20:00-21:30**    **Welcome Cocktail** - SCAI (Sorbonne Center for Artificial Intelligence)

# Thursday, July 22<sup>nd</sup>

## Day 1 of Author Sessions

Location: Centre  
International de  
Conférences  
Sorbonne Universités  
(Jussieu)

8:30-10:10

### ROOM A

#### SESSION: PROBABILITY & STATISTICS ON RIEMANNIAN MANIFOLDS

*Chairs: Xavier Pennec, Cyrus Mostajeran*

##### I From Bayesian inference to MCMC and convex optimisation in Hadamard manifolds

*Salem Said, Nicolas Le Bihan,  
and Jonathan H. Manton*

**Abstract.** The present work is motivated by the problem of Bayesian inference for Gaussian distributions in symmetric Hadamard spaces (that is, Hadamard manifolds which are also symmetric spaces). To investigate this problem, it introduces new tools for Markov Chain Monte Carlo, and convex optimisation: (1) it provides easy-to-verify sufficient conditions for the geometric ergodicity of an isotropic Metropolis-Hastings Markov chain, in a symmetric Hadamard space, (2) it shows how the Riemannian gradient descent method can achieve an exponential rate of convergence, when applied to a strongly convex function, on a Hadamard manifold. Using these tools, two Bayesian estimators, maximum-a-posteriori and minimum-mean-squares, are compared. When the underlying Hadamard manifold

is a space of constant negative curvature, they are found to be surprisingly close to each other. This leads to an open problem: are these two estimators, in fact, equal (assuming constant negative curvature) ?

##### I Gaussian distributions on Riemannian symmetric spaces in the large N limit

*Simon Heuveline, Salem Said,  
and Cyrus Mostajeran*

**Abstract.** We consider the challenging problem of computing normalization factors of Gaussian distributions on certain Riemannian symmetric spaces. In some cases, such as the space of Hermitian positive definite matrices or hyperbolic space, it is possible to compute them exactly using techniques from random matrix theory. However, in most cases which are important to applications, such as the space of symmetric positive definite (SPD) matrices or the Siegel domain, this is only possible numerically. Moreover, when we consider, for instance, high-dimensional SPD matrices, the known algorithms can become exceedingly slow. Motivated by notions from theoretical physics, we will discuss how to approximate these normalization factors in the large N limit: an approximation that gets increasingly better as the dimension of the underlying symmetric space (more precisely, its rank) gets larger. We will give formulas for leading order terms in the case of SPD matrices and related spaces. Furthermore, we will characterize the large N limit of the Siegel domain through a singular integral equation arising as a saddle-point equation.

##### I Finite Sample Smeariness on Spheres

*Benjamin Eltzner, Shayan Hundrieser,  
and Stephan Huckemann*

**Abstract.** Finite Sample Smeariness (FSS) has been recently discovered. It means that the distribution of sample Fréchet means of underlying rather unsuspecting random

variables can behave as if it were smeary for quite large regimes of finite sample sizes. In effect classical quantilebased statistical testing procedures do not preserve nominal size, they reject too often under the null hypothesis. Suitably designed bootstrap tests, however, amend for FSS. On the circle it has been known that arbitrarily sized FSS is possible, and that all distributions with a nonvanishing density feature FSS. These results are extended to spheres of arbitrary dimension. In particular all rotationally symmetric distributions, not necessarily supported on the entire sphere feature FSS of Type I. While on the circle there is also FSS of Type II it is conjectured that this is not possible on higher-dimensional spheres.



### I Smeariness Begets Finite Sample Smeariness

*Do Tran, Benjamin Eltzner, and Stephan Huckemann*

**Abstract.** Fréchet means are indispensable for nonparametric statistics on non-Euclidean spaces. For suitable random variables, in some sense, they “sense” topological and geometric structure. In particular, smeariness seems to indicate the presence of positive curvature. While smeariness may be considered more as an academical curiosity, occurring rarely, it has been recently demonstrated that finite sample smeariness (FSS) occurs regularly on circles, tori and spheres and affects a large class of typical probability distributions. FSS can be well described by the modulation measuring the quotient of rescaled expected sample mean variance and population variance. Under FSS it is larger than one – that is its value on Euclidean spaces – and this makes quantile based tests using tangent space approximations inapplicable. We show here that near smeary probability distributions there are always FSS probability distributions and as a first step towards the conjecture that all compact spaces feature smeary distributions, we establish directional smeariness under curvature bounds.

### I Online learning of Riemannian hidden Markov models in homogeneous Hadamard spaces ?

*Quinten Tupker, Salem Said, and Cyrus Mostajeran*

**Abstract.** Hidden Markov models with observations in a Euclidean space play an important role in signal and image processing. Previous work extending to models where observations lie in Riemannian manifolds based on the Baum-Welch algorithm suffered from high memory usage and slow speed. Here we present an algorithm that is online, more accurate, and offers dramatic improvements in speed and efficiency.

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## ROOM B

### SESSION: SUB-RIEMANNIAN GEOMETRY AND NEUROMATHEMATICS

*Chairs: Alessandro Sarti, Dario Prandi*

### I Submanifolds of fixed degree in graded manifolds for perceptual completion?

*G. Citti, G. Giovannardi, M. Ritoré, and A. Sarti*

**Abstract.** We extend to an Engel type structure a cortically inspired model of perceptual completion initially proposed in the Lie group of positions and orientations with a sub-Riemannian metric. According to this model, a given image is lifted in the group and completed by a minimal surface. The main obstacle in extending the model to a higher dimensional group, which can code also curvatures, is the lack of a good definition of codimension 2 minimal surface. We present here this notion, and describe an application to image completion.

**| An auditory cortex model for sound processing?**

*Rand Asswad, Ugo Boscain, Giuseppina Turco, Dario Prandi, and Ludovic Sacchelli*

**Abstract.** The reconstruction mechanisms built by the human auditory system during sound reconstruction are still a matter of debate. The purpose of this study is to refine the auditory cortex model introduced in [9], and inspired by the geometrical modelling of vision. The algorithm lifts the time-frequency representation of the degraded sound to the Heisenberg group, where it is reconstructed via a WilsonCowan integro-differential equation. Numerical experiments on a library of speech recordings are provided, showing the good reconstruction properties of the algorithm.



**| Conformal model of hypercolumns in V1 cortex and the Möbius group. Application to the visual stability problem**

*D.V. Alekseevsky*



**| Extremal controls for Duits car**

*Alexey Mashtakov*

**Abstract.** We study a time minimization problem for a model of a car that can move forward on a plane and turn in place. Trajectories of this system are used in image processing for the detection of salient lines. The problem is a modification of a well-known sub-Riemannian problem in the roto-translation group, where one of the controls is restricted to be non-negative. The problem is of interest in geometric control theory as a model example in which the set of admissible controls contains zero on the boundary. We apply a necessary optimality condition — Pontryagin maximum principle to obtain a Hamiltonian system for normal extremals. By analyzing the Hamiltonian system we show a technique to obtain a single explicit formula for extremal controls. We derive the extremal controls and express the extremal trajectories in quadratures.

**| Multi-shape registration with constrained deformations**

*Rosa Kowalewski and Barbara Gris*

**Abstract.** Based on a Sub-Riemannian framework, deformation modules provide a way of building large diffeomorphic deformations satisfying a given geometrical structure. This allows to incorporate prior knowledge about object deformations into the model as a means of regularisation [10]. However, diffeomorphic deformations can lead to deceiving results if the deformed object is composed of several shapes which are close to each other but require drastically different deformations. For the related Large Deformation Diffeomorphic Metric Mapping, which yields unstructured deformations, this issue was addressed in [2] introducing object boundary constraints. We develop a new registration problem, marrying the two frameworks to allow for different constrained deformations in different coupled shapes.

10:10–10:30

**Coffee break**  
+ Posters Session 1  
(see page 41)

10:30–12:10

**ROOM A**

SHAPES SPACES

*Chairs: Salem Said, Joan Glaunès*

**| Geodesics and Curvature of the Quotient-Affine Metrics on Full-Rank Correlation Matrices**

*Yann Thanwerdas and Xavier Pennec*

**Abstract.** Correlation matrices are used in many domains of neurosciences such as fMRI, EEG, MEG. However, statistical analyses often



rely on embeddings into a Euclidean space or into Symmetric Positive Definite matrices which do not provide intrinsic tools. The quotient-affine metric was recently introduced as the quotient of the affine-invariant metric on SPD matrices by the action of diagonal matrices. In this work, we provide most of the fundamental Riemannian operations of the quotient affine metric: the expression of the metric itself, the geodesics with initial tangent vector, the Levi-Civita connection and the curvature.

### Parallel Transport on Kendall Shape Spaces

*Nicolas Guigui, Elodie Maignant, Alain Trouvé, Xavier Pennec*

**Abstract.** Kendall shape spaces are a widely used framework for the statistical analysis of shape data arising from many domains, often requiring the parallel transport as a tool to normalise time series data or transport gradient in optimisation procedures. We present an implementation of the pole ladder, an algorithm to compute parallel transport based on geodesic parallelograms and compare it to methods by integration of the parallel transport ordinary differential equation.

### Diffusion Means and Heat Kernel on Manifolds

*Pernille Hansen, Benjamin Eitzner and Stefan Sommer*

**Abstract.** We introduce diffusion means as location statistics on manifold data spaces. A diffusion mean is defined as the starting point of an isotropic diffusion with a given diffusivity. They can therefore be defined on all spaces on which a Brownian motion can be defined and numerical calculation of sample diffusion means is possible on a variety of spaces using the heat kernel expansion. We present several classes of spaces, for which the heat kernel is known and sample diffusion means can therefore be calculated. As an example, we

investigate a classic data set from directional statistics, for which the sample Fréchet mean exhibits finite sample smeariness.

### A reduced parallel transport equation on Lie Groups with a left-invariant metric

*Nicolas Guigui and Xavier Pennec*

**Abstract.** This paper presents a derivation of the parallel transport equation expressed in the Lie algebra of a Lie group endowed with a left invariant metric. The use of this equation is exemplified on the group of rigid body motions  $SE(3)$ , using basic numerical integration schemes, and compared to the pole ladder algorithm. This results in a stable and efficient implementation of parallel transport. The implementation leverages the python package `geomstats` and is available online.

### Currents and K-functions for Fiber Point Processes

*Pernille EH. Hansen, Rasmus Waagepetersen, Anne Marie Svane, Jon Sparring, Hans JT. Stephensen, Stine Hasselholt, and Stefan Sommer*

**Abstract.** Analysis of images of sets of fibers such as myelin sheaths or skeletal muscles must account for both the spatial distribution of fibers and differences in fiber shape. This necessitates a combination of point process and shape analysis methodology. In this paper, we develop a K-function for fiber-valued point processes by embedding shapes as currents, thus equipping the point process domain with metric structure inherited from a reproducing kernel Hilbert space. We extend Ripley's K-function which measures deviations from spatial homogeneity of point processes to fiber data. The paper provides a theoretical account of the statistical foundation of the K-function, and we apply the K-function on simulated data and a data set of myelin sheaths. This includes a fiber data set consisting of myelin sheaths configurations at different debts.

**ROOM B**

**GEOMETRY OF QUANTUM STATES**

*Chairs: Florio Maria Ciaglia, Michel Berthier*

**I Q-Information Geometry of Systems**

*Christophe Corbier*

**Abstract.** This paper proposes a generalization of information geometry named Q-information geometry (Q-IG) related to systems described by parametric models, based R-Complex Finsler subspaces and leading to Q-spaces. A Q-space is a pair  $(M_{\tau}^n, |F_{\tau}^{\alpha, \beta}|)$  with  $M_{\tau}^n$  manifold of systems and  $|F_{\tau, x}|, x \in [0, n]$  a continuous function such that  $F_{\tau, x}^{\alpha, \beta} = F_{\tau, x}^{\alpha} + \sqrt{-1} F_{\tau, x}^{\beta}$  with a signature  $(+, +, -)$  where  $F_{\tau, x}^{\alpha}$  are  $\tau$ -Hermitian and  $\tau$ -non-Hermitian metrics, respectively. Experimental results are presented from a semi-finite acoustic waves guide.



**I Group actions and Monotone Metric Tensors: The qubit case**

*F. M. Ciaglia and F. Di Nocera*

**Abstract.** In recent works, a link between group actions and information metrics on the space of faithful quantum states has been highlighted in particular cases. In this contribution, we give a complete discussion of this instance for the particular case of the qubit.

**I Quantum Jensen-Shannon divergences between infinite-dimensional positive definite operators**

*Minh Ha Quang*

**Abstract.** This work studies a parametrized family of symmetric divergences on the set of Hermitian positive definite matrices which are defined using the  $\alpha$ -Tsallis entropy  $\forall \alpha \in \mathbb{R}$ . This family unifies in particular the Quantum Jensen-Shannon divergence, defined

using the von Neumann entropy, and the Jensen-Bregman Log-Det divergence. The divergences, along with their metric properties, are then generalized to the setting of positive definite trace class operators on an infinite-dimensional Hilbert space  $\forall \alpha \in \mathbb{R}$ . In the setting of reproducing kernel Hilbert space (RKHS) covariance operators, all divergences admit closed form formulas in terms of the corresponding kernel Gram matrices.



**I Towards a geometrization of quantum complexity and chaos**

*Davide Rattacaso, Patrizia Vitale, and Alioscia Hamma*

**Abstract.** In this paper, we show how the restriction of the Quantum Geometric Tensor to manifolds of states that can be generated through local interactions provides a new tool to understand the consequences of locality in physics. After a review of a first result in this context, consisting in a geometric out-of-equilibrium extension of the quantum phase transitions, we argue the opportunity and the usefulness to exploit the Quantum Geometric Tensor to geometrize quantum chaos and complexity.



**I Hunt's colorimetric effect from a quantum measurement viewpoint**

*Michel Berthier and Edoardo Provenzi*

**Abstract.** The main aim of this paper is to describe how the colorimetric phenomenon known as 'Hunt effect' can be understood in the quantum framework developed by the authors to model color perception of trichromatic observers. In classical colorimetry, the most common definition of the Hunt effect is that the colorfulness of a color increases with its luminance, however several other definitions are available. The need to establish a unique

and precise characterization of the features involved in the Hunt effect led us to propose novel mathematical definitions of colorimetric attributes. Within the newly established nomenclature, we show how the Hunt effect can be rigorously explained thanks to the duality between quantum states and effects.

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**12:10–13:10** Lunch Break (*not included*)

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**13:10–14:30**

## ROOM A

### GEOMETRIC AND STRUCTURE PRESERVING DISCRETIZATIONS

**Chairs:** *Alessandro Bravetti, Manuel de Leon*

#### I The Herglotz principle and vakonomic dynamics

*Manuel de Leon, Manuel Lainz and Miguel C. Muñoz – Lecanda*

**Abstract.** In this paper we study vakonomic dynamics on contact systems with nonlinear constraints. In order to obtain the dynamics, we consider a space of admissible paths, which are the ones tangent to a given submanifold. Then, we find the critical points of the Herglotz action on this space of paths. This dynamics can be also obtained through an extended Lagrangian, including Lagrange multiplier terms. This theory has important applications in optimal control theory for Herglotz control problems, in which the cost function is given implicitly, through an ODE, instead of by a definite integral. Indeed, these control problems can be considered as particular cases of vakonomic contact systems, and we can use the Lagrangian theory of contact systems in order to understand their symmetries and dynamics.

#### I Structure-preserving discretization of a coupled heat-wave system, as interconnected port-Hamiltonian systems

*Ghislain Haine and Denis Matignon*

**Abstract.** The heat-wave system is recast as the coupling of portHamiltonian subsystems (pHS), and discretized in a structure-preserving way by the Partitioned Finite Element Method (PFEM), [11]. Then, depending on the geometric configuration of the two domains, different asymptotic behaviours of the energy of the coupled system can be recovered at the numerical level, assessing the validity of the theoretical results of [22].



#### I Examples of symbolic and numerical computation in Poisson geometry

*M. Evangelista–Alvarado, J. C. Ruz–Pantaleón, and P. Suarez–Serrato*

**Abstract.** We developed two Python modules for symbolic and numerical computation in Poisson geometry. We explain how to use them through several illustrative examples, which include the computation of Hamiltonian and modular vector fields.



#### I New directions for contact integrators

*Alessandro Bravetti, Marcello Seri, and Federico Zadra*

**Abstract.** Contact integrators are a family of geometric numerical schemes which guarantee the conservation of the contact structure. In this work we review the construction of both the variational and Hamiltonian versions of these methods. We illustrate some of the advantages of geometric integration in the dissipative setting by focusing on models inspired by recent studies in celestial mechanics and cosmology.

**ROOM B**

**INFORMATION GEOMETRY  
IN PHYSICS**

**Chairs: Geert Verdoolaege, Jun Zhang**

**I Space-time thermo-mechanics  
for a material continuum**

*Emmanuelle Rouhaud, Richard Kerner, Israa Choucair, Roula El Nahas, Alexandre Charles, and Benoit Panicaud*

**Abstract.** The object of this article is to introduce a covariant description for a thermo-mechanical continuum. The conservation equations are written in this context and a constitutive model is derived for a reversible transformation. It is then possible to formulate a weak form of the problem to be solved for the finite transformations of a solid. A finite-element computation is next proposed using this approach for thermo-mechanical problems. The advantages of such a formulation are highlighted throughout the article.



**I Entropic dynamics yields reciprocal relations**

*Pedro Pessoa*

**Abstract.** Entropic dynamics is a framework for defining dynamical systems that is aligned with the principles of information theory. In an entropic dynamics model for motion on a statistical manifold, we find that the rate of changes for expected values is linear with respect to the gradient of entropy with reciprocal (symmetric) coefficients. Reciprocity principles have been useful in physics since Onsager. Here we show how the entropic dynamics reciprocity is a consequence of the information geometric structure of the exponential family, hence it is a general property that can be extended to a broader class of dynamical models.

**I Hamilton-Killing flows on statistical manifolds**

*Ariel Caticha*

**Abstract.** The symplectic and metric structures inherent to Quantum Mechanics (QM) have been discovered, and independently rediscovered by many authors. They have been extensively studied. Rather than taking the Hilbert space structure of QM as a given postulate, in this work we proceed in the opposite direction. Our goal is to derive or “reconstruct” the mathematical formalism of QM from more basic considerations of probability theory and information geometry. Our starting point is the recognition that probabilities are central to QM – they are not just a feature that is peculiar to quantum measurements. We derive QM as a form of dynamics on statistical manifolds, which for simplicity we restrict to a finite dimensional simplex. The important ingredients are two: on one hand the cotangent bundle associated to the simplex has a natural symplectic structure; and on the other hand, the cotangent bundle inherits its own natural metric structure from the information geometry of the underlying simplex. We impose a dynamics that preserves (in the sense of vanishing Lie derivatives) both the symplectic structure (a Hamilton flow) and the metric structure (a Killing flow). The result is a formalism in which the linearity of the Schrödinger equation, the emergence of a complex structure, Hilbert spaces, and the Born rule, are derived rather than postulated.



**I Geometric Hydrodynamics and Infinite-dimensional Newton's Equations**

*Boris Khesin, Gerard Misiolek and Klas Modin*

**Abstract.** We revisit the geodesic approach to ideal hydrodynamics and present a related geometric framework for Newton's equations on groups of diffeomorphisms

and spaces of probability densities. The latter setting is sufficiently general to include equations of compressible and incompressible fluid dynamics, magnetohydrodynamics, shallow water systems and equations of relativistic fluids. We illustrate this with selected examples, using the tools of infinite-dimensional information geometry, optimal transport, the Madelung transform, and the formalism of symplectic and Poisson reduction.

14:30–16:10

**ROOM A****LIE GROUP MACHINE LEARNING**

**Chairs:** Frédéric Barbaresco,  
Gery de Saxcé

**I Gibbs states on symplectic manifolds with symmetries**

*Charles-Michel Marle*

**Abstract.** In this paper,  $(M, \Omega)$  is a connected symplectic manifold on which a Lie group  $G$  acts by a Hamiltonian action, with a moment map  $J : M \rightarrow \mathfrak{g}^*$ . A short reminder of the definitions of statistical states, Gibbs states, entropy, generalized temperatures and associated thermodynamic functions is first given. Then several examples of such Gibbs states are presented, together with the associated thermodynamic functions. Examples are given too of symplectic manifolds on which a Lie group acts by a Hamiltonian action on which no Gibbs state built with a moment map of this action can exist. Most of these examples are obtained with the use of a remarkable isomorphism of a fully oriented three-dimensional Euclidean or pseudo-Euclidean vector space onto the Lie algebra of its Lie group of symmetries.

**I Gaussian Distributions on the Space of Symmetric Positive Definite Matrices from Souriau's Gibbs State for Siegel Domains by Coadjoint Orbit and Moment Map**

*Frédéric Barbaresco*

**Abstract.** We will introduce Gaussian distribution on the space of Symmetric Positive Definite (SPD) matrices, through Souriau's covariant Gibbs density by considering this space as the pure imaginary axis of the homogeneous Siegel upper half space where  $Sp(2n, \mathbb{R})/U(n)$  acts transitively. Gauss density of SPD matrices is computed through Souriau's moment map and coadjoint orbits. We will illustrate the model first for Poincaré unit disk, then Siegel unit disk and finally upper half space. For this example, we deduce Gauss density for SPD matrices.

**I On Gaussian Group Convex Models**

*Hideyuki Ishi*

**Abstract.** The Gaussian group model is a statistical model consisting of central normal distributions whose concentration matrices are of the form  $gg^T$ , where  $g$  is an element of a matrix group  $G$ . When the set of  $gg^T$  is convex in the vector space of real symmetric matrices, the set forms an affine homogeneous convex domain studied by Vinberg. In this case, we give the smallest number of samples such that the maximum likelihood estimator (MLE) of the parameter exists with probability one. Moreover, if the MLE exists, it is explicitly expressed as a rational function of the sample data.

**I Exponential-wrapped distributions on  $SL(2, \mathbb{C})$  and the Möbius group**

*Emmanuel Chevallier*

**Abstract.** In this paper we discuss the construction of probability distributions on the group  $SL(2, \mathbb{C})$  and the Möbius group using the

exponential map. In particular, we describe the injectivity and surjectivity domains of the exponential map and provide its Jacobian determinant. We also show that on  $SL(2, \mathbb{C})$  and the Möbius group, there are no isotropic distributions in the group sense.



**I Information Geometry and Hamiltonian Systems on Lie Groups**

*Daisuke Tarama and Jean-Pierre Francoise*

**Abstract.** The present paper deals with a class of left-invariant semidefinite metrics, called Fisher-Rao semi-definite metric, on Lie groups appearing in transformation models. It is assumed that a family of invariant probability density functions on the sample manifold is given and that these probability density functions are invariant under a smooth Lie group action. As have been studied by Barndorff-Nielsen and his coauthors, as well as Amari and his collaborators, Fisher-Rao semi-definite metric is naturally induced as a left-invariant semi-definite metric on the Lie group, which is regarded as the parameter space of the family of probability density functions. For a specific choice of family of probability density functions on compact semi-simple Lie group, the equation for the geodesic flow is derived through the Euler-Poincaré reduction. Certain perspectives are mentioned about the geodesic equation on the basis of its similarity with the Brockett double bracket equation and with the Euler-Arnol'd equation for a generalized free rigid body dynamics.

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**ROOM B**

**GEOMETRIC & SYMPLECTIC METHODS FOR HYDRODYNAMICAL MODELS**

**Chairs: Cesare Tronci, François Gay-Balmaz**

**I Multisymplectic variational integrators for fluid models with constraints**

*François Demoures and François Gay-Balmaz*

**Abstract.** We present a structure preserving discretization of the fundamental spacetime geometric structures of fluid mechanics in the Lagrangian description in 2D and 3D. Based on this, multisymplectic variational integrators are developed for barotropic and incompressible fluid models, which satisfy a discrete version of Noether theorem. We show how the geometric integrator can handle regular fluid motion in vacuum with free boundaries and constraints such as the impact against an obstacle of a fluid flowing on a surface. Our approach is applicable to a wide range of models including the Boussinesq and shallow water models, by appropriate choice of the Lagrangian.



**I Metriplectic Integrators for Dissipative Fluids**

*Michael Kraus*

**Abstract.** Many systems from fluid dynamics and plasma physics possess a so-called metriplectic structure, that is the equations are comprised of a conservative, Hamiltonian part, and a dissipative, metric part. Consequences of this structure are conservation of important quantities, such as mass, momentum and energy, and compatibility with the laws of thermodynamics, e.g., monotonic dissipation of entropy and existence of a unique equilibrium state. For simulations of such systems to deliver accurate and physically correct results, it is important to preserve these relations and conservation laws in the course of discretisation. This can be achieved most easily not by enforcing these properties directly, but by preserving the underlying abstract mathematical structure of the equations, namely their metriplectic structure. From that, the conservation of the aforementioned desirable properties follows automatically.

This paper describes a general and flexible framework for the construction of such metriplectic structure-preserving integrators, that facilitates the design of novel numerical methods for systems from fluid dynamics and plasma physics.

### I From quantum hydrodynamics to Koopman wavefunctions I

*François Gay-Balmaz and Cesare Tronci*

**Abstract.** Based on Koopman's theory of classical wavefunctions in phase space, we present the Koopman-van Hove (KvH) formulation of classical mechanics as well as some of its properties. In particular, we show how the associated classical Liouville density arises as a momentum map associated to the unitary action of strict contact transformations on classical wavefunctions. Upon applying the Madelung transform from quantum hydrodynamics in the new context, we show how the Koopman wavefunction picture is insufficient to reproduce arbitrary classical distributions. However, this problem is entirely overcome by resorting to von Neumann operators. Indeed, we show that the latter also allow for singular  $\delta$ -like profiles of the Liouville density, thereby reproducing point particles in phase space.

### I From quantum hydrodynamics to Koopman wavefunctions II?

*Cesare Tronci and François Gay-Balmaz*

**Abstract.** Based on the Koopman-van Hove (KvH) formulation of classical mechanics introduced in Part I, we formulate a Hamiltonian model for hybrid quantum-classical systems. This is obtained by writing the KvH wave equation for two classical particles and applying canonical quantization to one of them. We illustrate several geometric properties of the model regarding the associated

quantum, classical, and hybrid densities. After presenting the quantum-classical Madelung transform, the joint quantum-classical distribution is shown to arise as a momentum map for a unitary action naturally induced from the van Hove representation on the hybrid Hilbert space. While the quantum density matrix is positive by construction, no such result is currently available for the classical density. However, here we present a class of hybrid Hamiltonians whose flow preserves the sign of the classical density. Finally, we provide a simple closure model based on momentum map structures.

### I A Variational principle for the generalized Whitham equations

*Hamid Alemi Ardakani*

**Abstract.** This paper presents an averaged Lagrangian functional for the generalized Green-Naghdi equations for fluid sloshing in a container undergoing prescribed rigid-body motion in three dimensions. The new Green-Naghdi model derived by Alemi Ardakani (2021) has a form of potential vorticity conservation, which can be obtained from the particle-relabeling symmetry property of the Lagrangian (Miles & Salmon 1985; Dellar & Salmon 2005). In this paper, the assumption of zero-potential-vorticity flow is applied to the Green-Naghdi Lagrangian functional to derive a new set of Boussinesq-like evolution equations, which are a generalization of the Whitham equations for fluid sloshing in three dimensions.

16:10-16:30

Coffee Break  
+ Posters Session 1  
(see page 41)

16:30-18:10

**ROOM A**

**HARMONIC ANALYSIS ON LIE GROUPS**

**Chairs: Jean-Pierre Gazeau, Frédéric Barbaresco**

**I The Fisher information of curved exponential families and the elegant Kagan inequality**

*Gérard Letac*

**Abstract.** Curved exponential families are so general objects that they seem to have no interesting universal properties. However Abram Kagan [1] discovered in 1985 a remarkable inequality on their Fisher information. This note gives a modern presentation of this result and examples, comparing in particular noncentral and central Wishart distributions.

**I Continuous Wavelet transforms for vector-valued functions**

*Hideyuki Ishi and Kazuhide Oshiro*

**Abstract.** We consider continuous wavelet transforms associated to unitary representations of the semi-direct product of a vector group with a linear Lie group realized on the Hilbert spaces of square-integrable vector-valued functions. In particular, we give a concrete example of an admissible vector-valued function (vector field) for the 3-dimensional similitude group.

**I Entropy under disintegrations**

*Juan Pablo Vignéaux*

**Abstract.** We consider the differential entropy of probability measures absolutely

continuous with respect to a given  $\sigma$ -finite “reference” measure on an arbitrary measure space. We state the asymptotic equipartition property in this general case; the result is part of the folklore but our presentation is to some extent novel. Then we study a general framework under which such entropies satisfy a chain rule: disintegrations of measures. We give an asymptotic interpretation for conditional entropies in this case. Finally, we apply our result to Haar measures in canonical relation.

**I Koszul Information Geometry, Liouville-Mineur Integrable Systems and Moser Isospectral Deformation Method for Hermitian Positive-Definite Matrices**

*Frédéric Barbaresco*

**Abstract.** As soon as 1993, A. Fujiwara and Y. Nakamura have developed close links between Information Geometry and integrable system by studying dynamical systems on statistical models and completely integrable gradient systems on the manifolds of Gaussian and multinomial distributions. Recently, Jean-Pierre François has revisited this model and extended it to the Peakon systems. In parallel, in the framework of integrable dynamical systems and Moser isospectral deformation method, originated in the work of P. Lax, A.M. Perelomov has applied models for space of Positive Definite Hermitian matrices of order 2, considering this space as an homogeneous space. We conclude with links of Lax pairs with Souriau equation to compute coefficients of characteristic polynomial of a matrix. Main objective of this paper is to compose a synthesis of these researches on integrability and Lax pairs and their links with Information Geometry, to stimulate new developments at the interface of these disciplines.



## I Flapping Wing Coupled Dynamics in Lie Group Setting

*Zdravko Terze, Viktor Pandža, Marijan Andrić, and Dario Zlatar*

**Abstract.** In order to study dynamics of flapping wing moving in ambient fluid, the geometric modeling approach of fully coupled fluid-solid system is adopted, incorporating boundary integral method and time integrator in Lie group setting. If the fluid is assumed to be inviscid and incompressible, the configuration space of the fluid-solid system is reduced by eliminating fluid variables via symplectic reduction. Consequently, the equations of motion for the flapping wing are formulated without explicitly incorporating fluid variables, while effect of the fluid flow to the flapping wing overall dynamics is accounted for by the added mass effect only (computed by the boundary integral functions of the fluid density and the flow velocity potential). In order to describe additional viscous effects and include fluid vorticity and circulation in the system dynamics, vortex shedding mechanism is incorporated by enforcing Kutta conditions on the flapping wing sharp edges. In summary, presented approach exhibits significant computational advantages in comparison to the standard numerical procedures that - most commonly comprise inefficient discretization of the whole fluid domain. Most importantly, due to its 'mid-fidelity' computational efficiency, presented approach allows to be embedded in the 'automated' optimization procedure for the multi-criterial flapping wing flight design.

## ROOM B

### STATISTICAL MANIFOLD & HESSIAN INFORMATION GEOMETRY

**Chairs: Noemie Combe, Michel Nguiffo Boyom**

## I Canonical foliations of statistical manifolds with hyperbolic compact leaves

*Michel Boyom, Emmanuel Gnandi, and Stéphane Puechmorel*

**Abstract.** The sheaf of solutions  $\mathcal{J}\nabla$  of the Hessian equation on a gauge structure  $(M, \nabla)$  is a key ingredient for understanding important properties from the cohomological point of view. In this work, a canonical representation of the group associated by Lie third's theorem to the Lie algebra formed by the sections of  $\mathcal{J}\nabla$  is introduced. On the foliation it defines, a characterization of compact hyperbolic leaves is then obtained.



## I Open Problems In Global Analysis Structured Foliations And The Information Geometry.

*Michel Nguiffo Boyom*

**Abstract.** The global analysis deals with the algebraic topology (.eg. the formalism of Spencer) and the Sternberg geometry (eg. the formalism of Guillemin-Sternberg) of differential equations and their formal solutions. An immediate constant is the impacts of the global analysis on the quantitative global differential geometry. In this era the question of the existence of certain geometric structures is still open for lack of criteria of the integrability of the equations which define them. The purpose of this talk is to survey recent advances on some of these open questions. These advances basically concerned the geometry of Koszul and the symplectic geometry. The methods of the information geometry have inspired the introduction of new numerical invariants and new homological invariants. These new invariants have informed innovative approaches to those old open problems.



**I Curvature Inequalities And Simon's Type Formulas In Statistical Geometry**

*Barbara Opozda*

**Abstract.** We present some inequalities for curvatures and some applications of Simons' formulas to statistical structures.



**I Harmonicity of Conformally-Projectively Equivalent Statistical Manifolds and Conformal Statistical Submersions**

*Mahesh T V and K S Subrahmanian Moosath*

**Abstract.** In this paper, a condition for harmonicity of conformally-projectively equivalent statistical manifolds is given. Then conformal statistical submersion is introduced which is a generalization of the statistical submersion and obtained a necessary and sufficient condition for the harmonicity of conformal statistical submersion.



**I Algorithms for approximating means of semi-infinite quasi-Toeplitz matrices!**

*Dario A. Bini, Bruno Iannazzo, and Jie Meng*

**Abstract.** We provide algorithms for computing the Karcher mean of positive definite semi-infinite quasi-Toeplitz matrices. After showing that the power mean of quasi-Toeplitz matrices is a quasi-Toeplitz matrix, we obtain a first algorithm based on the fact that the Karcher mean is the limit of a family of power means. A second algorithm, that is shown to be more effective, is based on a generalization to the infinite dimensional case of a reliable algorithm for computing the Karcher mean in the finite-dimensional case. Numerical tests show that the Karcher mean of infinite-dimensional quasi-Toeplitz matrices can be effectively approximated with a finite number of parameters.

18:10-19:30

**ROOM A**

**GEOMETRIC MECHANICS**

**Chairs:** Gery de Saxcé, Frédéric Barbaresco

**I Archetypal Model of Entropy by Poisson Cohomology as Invariant Casimir Function in Coadjoint Representation and Geometric Fourier Heat Equation**

*Frédéric Barbaresco*

**Abstract.** In 1969, Jean-Marie Souriau introduced a "Lie Groups Thermodynamics" in the framework of Symplectic model of Statistical Mechanics. Based on this model, we will introduce a geometric characterization of Entropy as a generalized Casimir invariant function in coadjoint representation, where Souriau cocycle is a measure of the lack of equivariance of the moment mapping. The dual space of the Lie algebra foliates into coadjoint orbits that are also the level sets on the entropy that could be interpreted in the framework of Thermodynamics by the fact that motion remaining on these surfaces is non-dissipative, whereas motion transversal to these surfaces is dissipative. We will also explain the 2nd Principle in thermodynamics by definite positiveness of Souriau tensor extending the Koszul-Fisher metric from Information Geometry, and introduce a new geometric Fourier heat equation with the Souriau-Koszul-Fisher tensor. In conclusion, Entropy as Casimir function is characterized by Koszul Poisson Cohomology.



**I Bridge Simulation and Metric Estimation on Lie Groups**

*Mathias Højgaard Jensen, Sarang Joshi, and Stefan Sommer*

**Abstract.** We present a simulation scheme for simulating Brownian bridges on complete and connected Lie groups. We show how this simulation scheme leads to absolute

continuity of the Brownian bridge measure with respect to the guided process measure. This result generalizes the Euclidean result of Delyon and Hu to Lie groups. We present numerical results of the guided process in the Lie group  $SO(3)$ . In particular, we apply importance sampling to estimate the metric on  $SO(3)$  using an iterative maximum likelihood method.

### I Constructing the Hamiltonian from the behaviour of a dynamical system by proper symplectic decomposition

*Nima Shirafkan, Pierre Gosselet, Franz Bamer, Abdelbacet Oueslati, Bernd Markert, and G ery de Saxc e*

**Abstract.** The modal analysis is revisited through the symplectic formalism, what leads to two intertwined eigenproblems. Studying the properties of the solutions, we prove that they form a canonical basis. The method is general and works even if the Hamiltonian is not the sum of the potential and kinetic energies. On this ground, we want to address the following problem: data being given in the form of one or more structural evolutions, we want to construct an approximation of the Hamiltonian from a covariant snapshot matrix and to perform a symplectic decomposition. We prove the convergence properties of the method when the time discretization is refined. If the data cloud is not enough rich, we extract the principal component of the Hamiltonian corresponding to the leading modes, allowing to perform a model order reduction for very high dimension models. The method is illustrated by a numerical example.

### I Non-relativistic Limits of General Relativity

*Eric Bergshoeff, Johannes Lahnsteiner, Luca Romano, Jan Rosseel, and Ceyda Simsek*

**Abstract.** We discuss non-relativistic limits of general relativity. In particular, we define

a special fine-tuned non-relativistic limit, inspired by string theory, where the Einstein-Hilbert action has been supplemented by the kinetic term of a one-form gauge field. Taking the limit, a crucial cancellation takes place, in an expansion of the action in terms of powers of the velocity of light, between a leading divergence coming from the spin-connection squared term and another infinity that originates from the kinetic term of the one-form gauge field such that the finite invariant non-relativistic gravity action is given by the next subleading term. This non-relativistic action allows an underlying torsional Newton-Cartan geometry as opposed to the zero torsion Newton-Cartan geometry that follows from a more standard limit of General Relativity but it lacks the Poisson equation for the Newton potential. We will mention extensions of the model to include this Poisson equation.

## ROOM B

### DEFORMED ENTROPY, CROSS-ENTROPY, AND RELATIVE ENTROPY

**Chairs:** Ting-Kam Leonard Wong, L eonard Monsaigeon

### I A Primer on Alpha-Information Theory with Application to Leakage in Secrecy Systems

*Olivier Rioul*

**Abstract.** We give an informative review of the notions of R enyi's  $\alpha$ -entropy and  $\alpha$ -divergence, Arimoto's conditional  $\alpha$ -entropy, and Sibson's  $\alpha$ -information, with emphasis on the various relations between them. All these generalize Shannon's classical information measures corresponding to  $\alpha = 1$ . We present results on data processing inequalities and provide some new generalizations of the classical Fano's

inequality for any  $\alpha > 0$ . This enables one to  $\alpha$ -information as a information theoretic metric of leakage in secrecy systems. Such metric can bound the gain of an adversary in guessing some secret (any potentially random function of some sensitive dataset) from disclosed measurements, compared with the adversary's prior belief (without access to measurements).



**I Schrödinger encounters Fisher and Rao: a survey**

*Léonard Monsaïgeon and Dmitry Vorotnikov*

**Abstract.** In this short note we review the dynamical Schrödinger problem on the non-commutative Fisher-Rao space of positive semi-definite matrix-valued measures. The presentation is meant to be self-contained, and we discuss in particular connections with Gaussian optimal transport, entropy, and quantum Fisher information.



**I Projections with logarithmic divergences**

*Zhixu Tao and Ting-Kam Leonard Wong*

**Abstract.** In information geometry, generalized exponential families and statistical manifolds with curvature are under active investigation in recent years. In this paper we consider the statistical manifold induced by a logarithmic  $L(\alpha)$ -divergence which generalizes the Bregman divergence. It is known that such a manifold is dually projectively flat with constant negative sectional curvature, and is closely related to the  $F(\alpha)$ -family, a generalized exponential family introduced by the second author [17]. Our main result constructs a dual foliation of the statistical manifold, i.e., an orthogonal decomposition consisting of primal and dual autoparallel submanifolds. This decomposition, which can be naturally interpreted in terms of primal and dual projections with respect to the logarithmic

divergence, extends the dual foliation of a dually flat manifold studied by Amari [1]. As an application, we formulate a new  $L(\alpha)$ -PCA problem which generalizes the exponential family PCA [5].



**I Chernoff, Bhattacharyya, Rényi and Sharma-Mittal divergence analysis for Gaussian stationary ARMA processes**

*Eric Grivel*

**Abstract.** The purpose of this paper is first to derive the expressions of the Chernoff, Bhattacharyya, Rényi and Sharma-Mittal divergences when comparing two probability density functions of vectors storing  $k$  consecutive samples of Gaussian ARMA processes. Then, we analyze their behaviors when  $k$  increases and tends to infinity by using some results related to ARMA processes such as the Yule-Walker equations. Comments and illustrations are given.

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<b>20:00–22:00</b>	Gala Dinner Boat Cruise
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# Friday, July 23<sup>rd</sup>

## Day 2 of Author Sessions

Location: Centre  
International de  
Conférences Sorbonne  
Universités (Jussieu)

8:30–09:50

### ROOM A

#### TRANSPORT INFORMATION GEOMETRY

Chairs: Wuchen Li, Philippe Jacquet

#### | Wasserstein Statistics in One-dimensional Location-Scale Models

*Shun-ichi Amari and Takeru Matsuda*

**Abstract.** In this study, we analyze statistical inference based on the Wasserstein geometry in the case that the base space is one-dimensional. By using the location-scale model, we derive the W-estimator that explicitly minimizes the transportation cost from the empirical distribution to a statistical model and study its asymptotic behaviors. We show that the W-estimator is consistent and explicitly gives its asymptotic distribution by using the functional delta method. The W-estimator is Fisher efficient in the Gaussian case.

#### | Traditional and accelerated gradient descent for neural architecture search

*Nicolas García Trillos, Félix Morales, and Javier Morales*

**Abstract.** In this paper we introduce two algorithms for neural architecture search (NASGD and NASAGD) following the theoretical work by two of the authors [4] which used the geometric structure of optimal transport to introduce the conceptual basis for new notions of traditional and accelerated gradient descent algorithms for the optimization of a function on a semi-discrete space. Our algorithms, which use the network morphism framework introduced in [1] as a baseline, can analyze forty times as many architectures as the hill climbing methods [1, 10] while using the same computational resources and time and achieving comparable levels of accuracy. For example, using NASGD on CIFAR10, our method designs and trains networks with an error rate of 4.06 in only 12 hours on a single GPU.

#### | Recent developments on the MTW tensor

*Gabriel Khan and Jun Zhang*

**Abstract.** We survey some recent research related to the regularity theory of optimal transport and its associated geometry. We discuss recent progress and pose some open questions and a conjecture related to the MTW tensor, which provides a local obstruction to the smoothness of the Monge transport maps.

#### | Wasserstein Proximal of GANs

*Alex Tong Lin, Wuchen Li, Stanley Osher, and Guido Montúfar*

**Abstract.** We introduce a new method for training generative adversarial networks by applying the Wasserstein-2 metric proximal on the generators. The approach is based on Wasserstein information geometry. It defines a parameterization invariant natural gradient by pulling back optimal transport structures from probability

space to parameter space. We obtain easy-to-implement iterative regularizers for the parameter updates of implicit deep generative models. Our experiments demonstrate that this method improves the speed and stability of training in terms of wall-clock time and Fréchet Inception Distance.

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## ROOM B

### STATISTICS, INFORMATION, AND TOPOLOGY

**Chairs:** Pierre Baudot, Michel Nguiffo Boyom

#### I Information cohomology of classical vector-valued observables

*Juan Pablo Vigneaux*

**Abstract.** We provide here a novel algebraic characterization of two information measures associated with a vector-valued random variable, its differential entropy and the dimension of the underlying space, purely based on their recursive properties (the chain rule and the nullity-rank theorem, respectively). More precisely, we compute the information cohomology of Baudot and Bennequin with coefficients in a module of continuous probabilistic functionals over a category that mixes discrete observables and continuous vector-valued observables, characterizing completely the 1-cocycles; evaluated on continuous laws, these cocycles are linear combinations of the differential entropy and the dimension.

#### I On Marginal Estimation Algorithms - Belief Propagation as Diffusion

*Olivier Peltre*

#### I Towards a functorial description of quantum relative entropy

*Arthur J. Parzygnat*

**Abstract.** A Bayesian functorial characterization of the classical relative entropy (KL divergence) of finite probabilities was recently obtained by Baez and Fritz. This was then generalized to standard Borel spaces by Gagné and Panangaden. Here, we provide preliminary calculations suggesting that the finite-dimensional quantum (Umegaki) relative entropy might be characterized in a similar way. Namely, we explicitly prove that it defines an affine functor in the special case where the relative entropy is finite. A recent non-commutative disintegration theorem provides a key ingredient in this proof.

#### I Frobenius statistical manifolds & geometric invariants

*Noemie Combe, Philippe Combe, and Hanna Nencka*

**Abstract.** Using an algebraic approach, we prove that statistical manifolds, related to exponential families and with flat structure connections have a Frobenius manifold structure. This latter object, at the interplay of beautiful interactions between topology and quantum field theory, raises natural questions, concerning the existence of Gromov–Witten invariants for those statistical manifolds. We prove that an analog of Gromov–Witten invariants for those statistical manifolds (GWS) exists. Similarly to its original version, these new invariants have a geometric interpretation concerning intersection points of paraholomorphic curves. It also plays an important role in the learning process, since it determines whether a system has succeeded in learning or failed.

09:50–10:10

Coffee Break  
+ Posters Session 2  
(see page 43)

10:10–11:50

**ROOM A****GEOMETRIC DEEP LEARNING****Chairs:** Gabriel Peyré, Erick J. Bekkers**I SU(1,1) Equivariant Neural Networks and Application to Robust Toeplitz Hermitian Positive Definite Matrix Classification***Pierre-Yves Lagrave, Yann Cabanes,  
and Frédéric Barbaresco*

**Abstract.** In this paper, we propose a practical approach for building  $SU(1,1)$  equivariant neural networks to process data with support within the Poincaré disk  $D$ . By leveraging on the transitive action of  $SU(1,1)$  on  $D$ , we define an equivariant convolution operator on  $D$  and introduce a Helgason-Fourier analysis approach for its computation, that we compare with a conditional Monte-Carlo method. Finally, we illustrate the performance of such neural networks from both accuracy and robustness standpoints through the example of Toeplitz Hermitian Positive Definite (THPD) matrix classification in the context of radar clutter identification from the corresponding Doppler signals.

**I Iterative SE(3)-Transformers***Fabian B. Fuchs, Edward Wagstaff,  
Justas Dauparas, and Ingmar Posner*

**Abstract.** When manipulating three-dimensional data, it is possible to ensure that rotational and translational

symmetries are respected by applying so-called  $SE(3)$ -equivariant models. Protein structure prediction is a prominent example of a task which displays these symmetries. Recent work in this area has successfully made use of an  $SE(3)$ -equivariant model, applying an iterative  $SE(3)$ -equivariant attention mechanism. Motivated by this application, we implement an iterative version of the  $SE(3)$ -Transformer, an  $SE(3)$ -equivariant attention-based model for graph data. We address the additional complications which arise when applying the  $SE(3)$ -Transformer in an iterative fashion, compare the iterative and single-pass versions on a toy problem, and consider why an iterative model may be beneficial in some problem settings. We make the code for our implementation available to the community .

**I End-to-End Similarity Learning and Hierarchical Clustering for unfixed size datasets***Leonardo Gigli, Beatriz Marcotequi,  
Santiago Velasco-Forero*

**Abstract.** Hierarchical clustering (HC) is a powerful tool in data analysis since it allows discovering patterns in the observed data at different scales. Similarity-based HC methods take as input a fixed number of points and the matrix of pairwise similarities and output the dendrogram representing the nested partition. However, in some cases, the entire dataset cannot be known in advance and thus neither the relations between the points. In this paper, we consider the case in which we have a collection of realizations of a random distribution, and we want to extract a hierarchical clustering for each sample. The number of elements varies at each draw. Based on a continuous relaxation of Dasgupta's cost function, we propose to integrate a triplet loss function to Chami's formulation in order to learn an optimal

similarity function between the points to use to compute the optimal hierarchy. Two architectures are tested on four datasets as approximators of the similarity function. The results obtained are promising and the proposed method showed in many cases good robustness to noise and higher adaptability to different datasets compared with the classical approaches.



**Information theory and the embedding problem for Riemannian manifolds**

*Govind Menon*

**Abstract.** This paper provides an introduction to an information theoretic formulation of the embedding problem for Riemannian manifolds developed by the author. The method is illustrated with a new model for embedding finite metric spaces.



**cCorrGAN: Conditional Correlation GAN for Learning Empirical Conditional Distributions in the Elliptope**

*Gautier Marti, Victor Goubet, and Frank Nielsen*

**Abstract.** We propose a methodology to approximate conditional distributions in the elliptope of correlation matrices based on conditional generative adversarial networks. We illustrate the methodology with an application from quantitative finance: Monte Carlo simulations of correlated returns to compare risk-based portfolio construction methods. Finally, we discuss about current limitations and advocate for further exploration of the elliptope geometry to improve results.

**ROOM B**

**TOPOLOGICAL AND GEOMETRICAL STRUCTURES IN NEUROSCIENCES**

**Chairs: Pierre Baudot, Giovanni Petri**

**Topological Model of Neural Information Networks**

*Matilde Marcolli*

**Abstract.** This is a brief overview of an ongoing research project, involving topological models of neural information networks and the development of new versions of associated information measures that can be seen as possible alternatives to integrated information. Among the goals are a geometric modeling of a “space of qualia” and an associated mechanism that constructs and transforms representations from neural codes topologically. The more mathematical aspects of this project stem from the recent joint work of the author and Yuri Manin, [18], while the neuroscience modeling aspects are part of an ongoing collaboration of the author with Doris Tsao.



**On Information Links**

*Pierre Baudot*

**Abstract.** In a joint work with D. Bennequin [8], we suggested that the (negative) minima of the 3-way multivariate mutual information correspond to Borromean links, paving the way for providing probabilistic analogs of linking numbers. This short note generalizes the correspondence of the minima of  $k$  multivariate interaction information with  $k$  Brunnian links in the binary variable case. Following [16], the negativity of the associated K-L divergence of the joint probability law with its Kirkwood approximation implies an obstruction to local decomposition into lower order interactions than  $k$ , defining a local decomposition inconsistency that reverses Abramsky’s contextuality local-global relation [1]. Those



negative k-links provide a straightforward definition of collective emergence in complex k-body interacting systems or dataset.



### I Betti Curves of Rank One Symmetric Matrices

*Carina Curto, Joshua Paik, and Igor Rivin*

**Abstract.** Betti curves of symmetric matrices were introduced in [3] as a new class of matrix invariants that depend only on the relative ordering of matrix entries. These invariants are computed using persistent homology, and can be used to detect underlying structure in biological data that may otherwise be obscured by monotone nonlinearities. Here we prove three theorems that fully characterize the Betti curves of rank 1 symmetric matrices. We then illustrate how these Betti curve signatures arise in natural data obtained from calcium imaging of neural activity in zebrafish.



### I Algebraic Homotopy Interleaving Distance

*Nicolas Berkouk*

**Abstract.** The theory of persistence, which arises from topological data analysis, has been intensively studied in the one-parameter case both theoretically and in its applications. However, its extension to the multiparameter case raises numerous difficulties, where it has been proven that no barcode-like decomposition exists.

To tackle this problem, algebraic invariants have been proposed to summarize multi-parameter persistence modules, adapting classical ideas from commutative algebra and algebraic geometry to this context. Nevertheless, the crucial question of the stability of these invariants has raised little attention so far, and many of the proposed invariants do not satisfy a naive form of stability.

In this paper, we equip the homotopy and the derived category of multiparameter persistence modules with an appropriate interleaving distance. We prove that resolution functors are always isometric with respect to this distance. As an application, this explains why the graded-Betti numbers of a persistence module do not satisfy a naive form of stability. This opens the door to performing homological algebra operations while keeping track of stability. We believe this approach can lead to the definition of new stable invariants for multi-parameter persistence, and to new computable lower bounds for the interleaving distance (which has been recently shown to be NP-hard to compute in [2]).



### I A Python hands-on tutorial on network and topological neuroscience

*Eduarda Gervini Zampieri Centeno, Giulia Moreni, Chris Vriend, Linda Douw, Fernando Antônio Nóbrega Santos*

**Abstract.** Network neuroscience has been trying to uncover brain functioning through the prism of connectivity, and graph theory has been an essential branch in the investigation of brain networks. Recently, topological data analysis has gained more attention as an alternative framework by providing a set of metrics that go beyond pairwise connections and offer improved robustness against noise. Here, our goal is to provide an easy-to-grasp theoretical and computational tutorial to explore neuroimaging data using these frameworks, with the aim to facilitate their accessibility, data visualisation, and comprehension for newcomers to the field. We provide a concise (and by no means complete) theoretical overview of the two frameworks and a computational guide on the computation of both well-established and newer metrics using a publicly available resting-state functional magnetic resonance imaging dataset. Moreover, we have developed a pipeline for three-dimensional (3-D) visualisation of high order interactions in brain networks.

11:50-13:00

Lunch Break (*not included*)

13:10-14:40

**ROOM A**

**COMPUTATIONAL INFORMATION GEOMETRY 1**

**Chairs: Frank Nielsen, Clément Gauchy**

**Computing statistical divergences with sigma points**

*Frank Nielsen and Richard Nock*

**Abstract.** We show how to bypass the integral calculation and express the Kullback-Leibler divergence between any two densities of an exponential family using a finite sum of logarithms of density ratio evaluated at sigma points. This result allows us to characterize the exact error of Monte Carlo estimators. We then extend the sigma point method to the calculations of q-divergences between densities of a deformed q exponential family.

**Remarks on Laplacian of graphical models in various graphs**

*Tomasz Skalski*

**Abstract.** This paper discusses the connection between inverses of graph Laplacians with the diagonal increased by a positive vector and the covariance matrices of associated Gaussian graphical models. Afterwards we study the way of modification of a graph Laplacian on the case of daisy graphs.

**Classification in the Siegel Space for Vectorial Autoregressive Data**

*Yann Cabanes and Frank Nielsen*

**Abstract.** We introduce new geometrical tools to cluster data in the Siegel space. We give the expression of the Riemannian logarithm and exponential maps in the Siegel disk. These new tools help us to perform classification algorithms in the Siegel disk. We also give the expression of the sectional curvature in the Siegel disk. The sectional curvatures are negative or equal to zero, and therefore the curvature of the Siegel disk is non-positive. This result proves the convergence of the gradient descent performed when computing the mean of a set of matrix points in the Siegel disk.

**Information Metrics for Phylogenetic Trees via Distributions of Discrete and Continuous Characters**

*Maryam K. Garba, Tom M. W. Nye, Jonas Lueg, and Stephan F. Huckemann*

**Abstract.** A wide range of metrics between phylogenetic trees are used in evolutionary molecular biology. They are typically based directly on the branching patterns and edge lengths represented by the trees. Metrics have recently been proposed which are based on the information content of distributions of genetic characters induced by the trees. We first show how these metrics lead to a change to the topology of the underlying tree space. Next we show via computational methods that the metrics are stable under changes to the Markov process used to generate characters, at least in the case of 5 taxa. As a result, a Gaussian process defined over the edges of trees can be used to compute the metrics, leading to a substantial computational efficiency over DNA nucleotide-valued Markov process models.

**Wald Space for Phylogenetic Trees**

*Jonas Lueg, Maryam K. Garba, Tom M. W. Nye and Stephan F. Huckemann*

**Abstract.** The recently introduced wald space models phylogenetic trees from an evolutionary

perspective. We show that it is a stratified space and propose algorithms to compute geodesics. In application we compute a Fréchet mean of three trees of different topologies that is fully resolved, unlike in BHV-space. Both, preliminary results on geodesics and on means suggest that wald space features less stickiness than BHVspace, making it an alternative model for statistical investigations.

## ROOM B

### MANIFOLDS & OPTIMIZATION

**Chairs:** *Stéphanie Jehan-Besson, Bin Gao*

#### **I Efficient Quasi-Geodesics on the Stiefel Manifold**

*Thomas Bendokat and Ralf Zimmermann*

**Abstract.** Solving the so-called geodesic endpoint problem, i.e., finding a geodesic that connects two given points on a manifold, is at the basis of virtually all data processing operations, including averaging, clustering, interpolation and optimization. On the Stiefel manifold of orthonormal frames, this problem is computationally involved. A remedy is to use quasi-geodesics as a replacement for the Riemannian geodesics. Quasigeodesics feature constant speed and covariant acceleration with constant (but possibly non-zero) norm. For a well-known type of quasi-geodesics, we derive a new representation that is suited for large-scale computations. Moreover, we introduce a new kind of quasi-geodesics that turns out to be much closer to the Riemannian geodesics.

#### **I Optimization of a shape metric based on information theory applied to segmentation fusion and evaluation in multimodal MRI for DIPG tumor analysis**

*S. Jehan-Besson, R. Clouard, N. Boddart, J. Grill, and F. Frouin*

**Abstract.** In medical imaging, the construction of a reference shape from a set of segmentation results from different algorithms or image modalities is an important issue when dealing with the evaluation of segmentation without knowing the gold standard or when an evaluation of the inter or intra expert variability is needed. It is also interesting to build this consensus shape to merge the results obtained for the same target object from automatic or semi-automatic segmentation methods. In this paper, to deal with both segmentation fusion and evaluation, we propose to define such a «mutual shape» as the optimum of a criterion using both the mutual information and the joint entropy of the segmentation methods. This energy criterion is justified using the similarities between quantities of information theory and area measures and is presented in a continuous variational framework. We investigate the applicability of our framework for the fusion and evaluation of segmentation methods in multimodal MR images of diffuse intrinsic pontine glioma (DIPG).

#### **I Metamorphic image registration using a semi-Lagrangian scheme**

*Anton François, Pietro Gori, and Joan Glauènes*

**Abstract.** In this paper, we propose an implementation of both Large Deformation Diffeomorphic Metric Mapping (LDDMM) and Metamorphosis image registration using a semi-Lagrangian scheme for geodesic shooting. We propose to solve both problems as an inexact matching providing a single and unifying cost function. We demonstrate that for image registration the use of a semi-Lagrangian scheme is more stable than a standard Eulerian scheme. Our GPU implementation is based on PyTorch, which greatly simplifies and accelerates the computations thanks to its powerful automatic differentiation engine. It will be freely available at <https://github.com/antonfrancois/Demeter-metamorphosis>.

**I Geometry of the symplectic Stiefel manifold endowed with the Euclidean metric**

*Bin Gao, Nguyen Thanh Son, P.-A. Absil, and Tatjana Stykel*

**Abstract.** The symplectic Stiefel manifold, denoted by  $Sp(2p, 2n)$ , is the set of linear symplectic maps between the standard symplectic spaces  $R^{2p}$  and  $R^{2n}$ . When  $p = n$ , it reduces to the well-known set of  $2n \times 2n$  symplectic matrices. We study the Riemannian geometry of this manifold viewed as a Riemannian submanifold of the Euclidean space  $R^{2n \times 2p}$ . The corresponding normal space and projections onto the tangent and normal spaces are investigated. Moreover, we consider optimization problems on the symplectic Stiefel manifold. We obtain the expression of the Riemannian gradient with respect to the Euclidean metric, which is then used in optimization algorithms. Numerical experiments on the nearest symplectic matrix problem and the symplectic eigenvalue problem illustrate the effectiveness of Euclidean-based algorithms.

statistical experiments, statisticians can at least partially design the data-generating process to obtain a high estimation performance. In this paper, a necessary condition for a semiparametrically efficient experimental design is proposed. A formula to determine the efficient distribution of input variables is derived. An application to the optimal bid design problem of contingent valuation survey experiments is presented.



**I Parametrisation Independence of the Natural Gradient in Overparametrised Systems**

*Jesse van Oostrum and Nihat Ay*

**Abstract.** In this paper we study the natural gradient method for overparameterised systems. This method is based on the natural gradient field which is invariant with respect to coordinate transformations. One calculates the natural gradient of a function on the manifold by multiplying the ordinary gradient of the function by the inverse of the Fisher Information Matrix (FIM). In overparametrised models, the FIM is degenerate and therefore one needs to use a generalised inverse. We show explicitly that using a generalised inverse, and in particular the MoorePenrose inverse, does not affect the parametrisation independence of the natural gradient. Furthermore, we show that for singular points on the manifold the parametrisation independence is not even guaranteed for non-overparametrised models.



14:40-16:20

**ROOM A**

**COMPUTATIONAL INFORMATION GEOMETRY 2**

**Chairs: Giovanni Pistone, Goffredo Chirco**

**I Necessary Condition for Semiparametric Efficiency of Experimental Designs**

*Hisatoshi Tanaka*

**Abstract.** The efficiency of estimation depends not only on the estimation method but also on the distribution of data. In

**I Properties of nonlinear diffusion equations on networks and their geometric aspects**

*Atsumi Ohara and Xiaoyan Zhang*

**Abstract.** We consider a fairly wide class of nonlinear diffusion equations on

networks, and derive several common and basic behaviors of solutions to them. Further, we demonstrate that the Legendre structure can be naturally introduced for such a class of dynamical systems, and discuss their information geometric aspects.

### I Rényi Relative Entropy from Homogeneous Kullback-Leibler Divergence Lagrangian

*Goffredo Chirco*

**Abstract.** We study the homogeneous extension of the Kullback-Leibler divergence associated to a covariant variational problem on the statistical bundle. We assume a finite sample space. We show how such a divergence can be interpreted as a Finsler metric on an extended statistical bundle, where the time and the time score are understood as extra random functions defining the model. We find a relation between the homogeneous generalisation of the Kullback-Leibler divergence and the Rényi relative entropy, the Rényi parameter being associated to the time-reparametrization lapse of the model. We investigate such intriguing relation with an eye to applications in physics and quantum information theory.

### I Statistical bundle of the transport model

*Giovanni Pistone*

**Abstract.** We discuss the statistical bundle of the manifold of two variate strictly positive probability functions with given marginals. The fiber associated to each coupling turns out to be the vector space of interactions in the ANOVA decomposition with respect to the given weight. In this setting, we derive the form of the gradient flow equation for the Kantorovich optimal transport problem.

## ROOM B

### DIVERGENCE STATISTICS

**Chairs:** Michel Broniatowski, Wolfgang Stummer

#### I On f-divergences between Cauchy distributions

*Frank Nielsen and Kazuki Okamura*

**Abstract.** We prove that the f-divergences between univariate Cauchy distributions are always symmetric and can be expressed as a function of the chi-squared divergence. We show that this property does not hold anymore for multivariate Cauchy distributions.

#### I Transport information Hessian distances

*Wuchen Li*

**Abstract.** We formulate closed-form Hessian distances of information entropies in one dimensional probability density space embedded with the L2-Wasserstein metric. Some analytical examples are provided.

#### I Minimization with respect to divergences and applications

*Pierre Bertrand, Michel Broniatowski, and Jean-François Marcotorchino*

**Abstract.** We apply divergences to project a prior guess discrete probability law on pq elements towards a subspace defined by fixed margins constraints  $\mu$  and  $\nu$  on p and q elements respectively. We justify why the Kullback-Leibler and the Chi-square divergences are two canonical choices based on a 1991 work of Imre Csiszár. Besides, we interpret the so-called indetermination resulting from the second divergence as a construction to reduce couple matchings. Eventually, we demonstrate how both resulting probabilities arise in two information

theory applications: guessing problem and task partitioning where some optimization remains to minimize a divergence projection.



**I Optimal transport with some directed distances**

*Wolfgang Stummer*

**Abstract.** We present a toolkit of directed distances between quantile functions. By employing this, we solve some new optimal transport (OT) problems which e.g. considerably flexibilize some prominent OTs expressed through Wasserstein distances.



**I Robust Empirical Likelihood**

*Amor Keziou and Aida Toma*

**Abstract.** In this paper, we present a robust version of the empirical likelihood estimator for semiparametric moment condition models. This estimator is obtained by minimizing the modified Kullback-Leibler divergence, in its dual form, using truncated orthogonality functions. Some asymptotic properties regarding the limit laws of the estimators are stated.



**I Mind2Mind : Transfer Learning for GANs**

*Yael Fregier and Jean-Baptiste Gouray*

**Abstract.** Training generative adversarial networks (GANs) on high quality (HQ) images involves important computing resources. This requirement represents a bottleneck for the development of applications of GANs. We propose a transfer learning technique for GANs that significantly reduces training time. Our approach consists of freezing the low-level layers of both the critic and generator of the original GAN. We assume an auto-encoder constraint to ensure the compatibility of the internal representations of the critic and the generator. This assumption explains the gain in training time as it enables us to bypass the low-level layers during the forward and backward passes. We compare our method to baselines and observe a significant acceleration of the training. It can reach two orders of magnitude on HQ datasets when compared with StyleGAN. We provide a theorem, rigorously proven within the framework of optimal transport, ensuring the convergence of the learning of the transferred GAN. We moreover provide a precise bound for the convergence of the training in terms of the distance between the source and target dataset.

16:20-16:40

Coffee Break  
+ Posters Session 2  
(see page 43)

16:40-18:00

**ROOM A**

**OPTIMAL TRANSPORT & LEARNING**

**Chairs:** Yaël Frégier, Nicolas Garcia Trillos

**I Fast and asymptotic steering to a steady state for networks flows**

*Yongxin Chen, Tryphon Georgiou, and Michele Pavon*

**Abstract.** We study the problem of optimally steering a network flow to a desired steady state, such as the Boltzmann distribution with a lower temperature, both in finite time and asymptotically. In the infinite horizon case, the problem is formulated as constrained minimization of the relative entropy rate. In such a case, we find that, if the prior is reversible, so is the solution.

## I Geometry of Outdoor Virus Avoidance in Cities

*Philippe Jacquet and Liubov Tupikina*

**Abstract.** We present low complexity and efficient path selection algorithms for limiting the exposure to pandemic virus during and between lock-downs. The average outdoor exposure rate reduction is around 3.



## I A Particle-Evolving method for approximating the Optimal Transport plan

*Shu Liu, Haodong Sun, and Hongyuan Zha*

**Abstract.** We propose an innovative algorithm that iteratively evolves a particle system to approximate the sample-wised Optimal Transport plan for given continuous probability densities. Our algorithm is proposed via the gradient flow of certain functional derived from the Entropy Transport Problem constrained on probability space, which can be understood as a relaxed Optimal Transport problem. We realize our computation by designing and evolving the corresponding interacting particle system. We present theoretical analysis as well as numerical verifications to our method.

**Abstract.** Aim of this paper is to take the first steps in the study of the Schrödinger problem for lattice gases (SPLG), which we formulate relying on classical results in large deviations theory. Our main contributions are a dynamical characterization of optimizers through a coupled system of PDEs and a precise study of the evolution and convexity of the quasi-potential along Schrödinger bridges. In particular, our computations show that, although SPLG does not admit a variational interpretation through Otto calculus, the fundamental geometric properties of the classical Schrödinger problem for independent particles still admit a natural generalization. These observations motivate the development of a Riemannian calculus on the space of probability measures associated with the class of geodesic distances studied in [3]. All our computations are formal, further efforts are needed to turn them into rigorous results.



## I A variational perspective on the thermodynamics of non-isothermal reacting open systems

*François Gay-Balmaz and Hiroaki Yoshimura*

**Abstract.** We review the variational formulation of nonequilibrium thermodynamics as an extension of the Hamilton principle and the Lagranged'Alembert principle of classical mechanics. We focus on the case of open systems that include the power exchange due to heat and matter transfer, with special emphasis on reacting systems which are very important in biological science.



## I On the Thermodynamic Interpretation of Deep Learning Systems

*R. Fioresi, F. Faglioni, F. Morri, and L. Squadrani*

**Abstract.** In the study of time evolution of the parameters in Deep Learning systems, subject to optimization via SGD (stochastic gradient descent), temperature, entropy and other thermodynamic notions are commonly

## ROOM B

### GEOMETRIC STRUCTURES IN THERMODYNAMICS AND STATISTICAL PHYSICS

**Chairs:** Hiroaki Yoshimura,  
François Gay-Balmaz

#### I Schrödinger problem for lattice gases: a heuristic point of view

*Alberto Chiarini, Giovanni Conforti,  
Luca Tamanini*

employed to exploit the Boltzmann formalism. We show that, in simulations on popular databases (CIFAR10, MNIST), such simplified models appear inadequate: different regions in the parameter space exhibit significantly different temperatures and no elementary function expresses the temperature in terms of learning rate and batch size, as commonly assumed. This suggests a more conceptual approach involving contact dynamics and Lie Group Thermodynamics.



**| Dirac structures in thermodynamics of non-simple systems**

*Hiroaki Yoshimura and François Gay-Balmaz*

**Abstract.** We present the Dirac structures and the associated Dirac system formulations

for non-simple thermodynamic systems by focusing upon the cases that include irreversible processes due to friction and heat conduction. These systems are called non-simple since they involve several entropy variables. We review the variational formulation of the evolution equations of such non-simple systems. Then, based on this, we clarify that there exists a Dirac structure on the Pontryagin bundle over a thermodynamic configuration space and we develop the Dirac dynamical formulation of such non-simple systems. The approach is illustrated with the example of an adiabatic piston.

18:00–18:30

GSJ'21 Closure

## | Poster Sessions

Poster Session 1	
<b>Clustering Schemes on the Torus with Application to RNA Clashes</b>	Henrik Wiechers, Benjamin Eltzner, Stephan F. Huckemann and Kanti V. Mardî
<b>Separating Point Patterns for Fingerprints</b>	Johannes Wieditz, Yvo Pokern, Dominic Schuhmacher, and Stephan Huckemann
<b>Statistical manifolds &amp; Hidden symmetries</b>	Noemie Combe, Philippe Combe, and Hanna Nencka
<b>Nonparametric Estimation of Probability Density Functions with Gaussian Processes</b>	T. T. Tran, C. Samir, and A. Fradi
<b>On Information (pseudo) Metric</b>	Pierre Baudot
<b>Nonparametric Regression on the Compact Stiefel Manifold</b>	Ines Adouani and Chafik Samir
<b>On Riemannian Stochastic Approximation Schemes with Fixed Step-Size</b>	Alain Durmus, Pablo Jiménez, Éric Moulines, Salem Said



## I Clustering Schemes on the Torus with Application to RNA Clashes

*Henrik Wiechers, Benjamin Eltzner, Stephan F. Huckemann and Kanti V. Mardia*

**Abstract.** Molecular structures of RNA molecules reconstructed from X-ray crystallography frequently contain errors. Motivated by this problem we examine clustering on a torus since RNA shapes can be described by dihedral angles. A previously developed clustering method for torus data involves two tuning parameters and we assess clustering results for different parameter values in relation to the problem of so-called RNA clashes. This clustering problem is part of the dynamically evolving field of statistics on manifolds. Statistical problems on the torus highlight general challenges for statistics on manifolds. Therefore, the torus PCA and clustering methods we propose make an important contribution to directional statistics and statistics on manifolds in general.

## I Separating Point Patterns for Fingerprints

*Johannes Wieditz, Yvo Pokern, Dominic Schuhmacher, and Stephan Huckemann*

**Abstract.** In a proof of concept, as an alternative to an MCMC driven separation of a homogeneous point process from an inhomogeneous hard core Strauss process for application in fingerprint analysis we propose and solve a pseudo-likelihood scheme. While the MCMC method can only be applied outside the singularities of fingerprint orientation fields, here we illustrate an approach taking the entire fingerprint into account leading to a separation of the minutiae point process into necessary minutiae, induced by orientation field and inter-ridge distance divergence and characteristic minutiae, due to individual biological randomness.

## I Statistical manifolds & Hidden symmetries

*Noemie Combe, Philippe Combe, and Hanna Nencka*

**Abstract..** In this paper, we investigate the manifold of probability distributions in the light of F-manifolds, and show that it is a paracomplex manifold. It decomposes into a pair of totally geodesic submanifolds, containing a pair of flat connections. We prove that the pair of pseudo Riemannian submanifolds are symmetric to each other with respect to Pierce mirror.



## I Nonparametric Estimation of Probability Density Functions with Gaussian Processes

*T. T. Tran, C. Samir, and A. Fradi*

**Abstract.** Gaussian processes are widely used in statistical modeling. In this paper, we propose to model the square root-density functions with Gaussian processes in a regression model. We consider the Mercer's representation of several classes of covariance functions from which we are able to find the corresponding eigenvalues and eigenfunctions. The practical interest of the proposed method is illustrated with several experiments.

## I On Information (pseudo) Metric

*Pierre Baudot*

**Abstract.** This short note revisits information metric, underlining that it is a pseudo metric on manifolds of observables (random variables), rather than as usual on probability laws. Geodesics are characterized in terms of their boundaries and conditional independence condition. Pythagorean theorem is given, providing in special cases potentially interesting natural integer triplets. This metric is computed for illustration on Diabetes dataset using infotopo package.

**I Nonparametric Regression on the Compact Stiefel Manifold**

*Ines Adouani and Chafik Samir*

**Abstract.** This paper addresses the problem of regularized non-linear regression on the Stiefel manifold when only few observations

are available. In particular, we consider the problem of estimating missing data using a continuous and smooth temporal path to overcome the discrete nature of observations. We give all numerical details and we show how this solution is geometrically simpler, extensible and can be transposed to other manifolds and applications.

Poster Session 2	
Application of information geometry in uncertainty quantification of computer codes	Clément Gauchy and Jérôme Stenger
On Geometry of Stein Goodness-of-fit Testing	Wenkai Xu and Takeru Matsuda
Simulation of Quantum Noise for Coherent States	Marco Frasca and Alfonso Farina
On the Quantization of the Uniform Source on the n-Sphere (n = 1,2,3)	Stéphane Ragot and Mohamed Yaoumi
Flattening Multiparameter Hierarchical Clustering Functors	Dan Shiebler
Learning the GENERIC evolution	Martin Sípka and Michal Pavelka

**I Application of information geometry in uncertainty quantification of computer codes**

*Clément Gauchy and Jérôme Stenger*

**Abstract.** Robust analysis consists in assessing the variation of a quantity of interest from a computer code relatively to the uncertain input probabilistic model. In an uncertainty quantification study, each parameter of the numerical model is commonly modeled as a random variable with known probability distribution. As a result the model output is also random and the statistician is able to perform various tasks

such as reliability assessment. However, in some cases the probability distributions of the input parameters are not fully identified and also tainted with uncertainties. Robustness studies is an emerging field in the uncertainty quantification domain, it aims at quantifying the impact of the uncertain input probability distributions on the quantity of interest of the computer model output (e.g a quantile). In this paper, we show how information geometry provides an elegant way of perturbing the input probability distribution, whenever an uncertainty affects its choice. To do so, we compute Fisher spheres in the manifold of the probability distributions associated to the Fisher-Rao metric. This

perturbation definition is then used to compute quantile-oriented robustness indices, the Perturbed-Law based sensitivity Indices (PLI).

### | On Geometry of Stein Goodness-of-fit Testing

*Wenkai Xu and Takeru Matsuda*

**Abstract.** We address the problem of goodness-of-fit testing for data on Riemannian manifolds. Due to the different topologies, the standard statistical procedures for multivariate data in  $\mathbb{R}^d$  are not applicable to such data. We derive the Stein operators on Riemannian manifolds and their corresponding goodness-of-fit testing procedures for general distributions including those with an intractable normalization constant. We discuss the geometric interpretations of different Stein operators.

### | Simulation of Quantum Noise for Coherent States

*Marco Frasca and Alfonso Farina*

**Abstract.** We show that, using standard stochastic processes, we are able to give a representation of the quantum noise of coherent states. This can be extended to any kind of known quantum process derivable by the Schrödinger equation. The corresponding Green function can also be computed. We confirm our result by a Monte Carlo simulation.

### | On the Quantization of the Uniform Source on the $n$ -Sphere ( $n = 1, 2, 3$ )

*Stéphane Ragot and Mohamed Yaoumi*

**Abstract.** We address in this article the quantization of the uniform source on the  $n$ -sphere  $s_n$  ( $n = 1, 2, 3$ ). We extend previous results on the rate/distortion (R/D) function for  $s_1$  and  $s_2$  and the mean squared error (MSE) from Harremoës to give explicit

R/D bounds for  $s_2$  and  $s_3$ . We present experimental R/D results for selected spherical codes on  $s_n$  ( $n = 1, 2, 3$ ). The performance gap at high bit rates between spherical codes on  $s_n$  and the R/D bound is compared empirically with the optimal lattice granular gain in dimension  $n$ . We also discuss the coding of rotations in  $SO(n)$  ( $n = 2, 3, 4$ ), which is formulated as an extension of the spherical quantization problem.

### | Flattening Multiparameter Hierarchical Clustering Functors

*Dan Shiebler*

**Abstract.** We bring together topological data analysis, applied category theory, and machine learning to study multiparameter hierarchical clustering. We begin by introducing a procedure for flattening multiparameter hierarchical clusterings. We demonstrate that this procedure is a functor from a category of multiparameter hierarchical partitions to a category of binary integer programs. We also include empirical results demonstrating its effectiveness. Next, we introduce a Bayesian update algorithm for learning clustering parameters from data. We demonstrate that the composition of this algorithm with our flattening procedure satisfies a consistency property.

### | Learning the GENERIC evolution

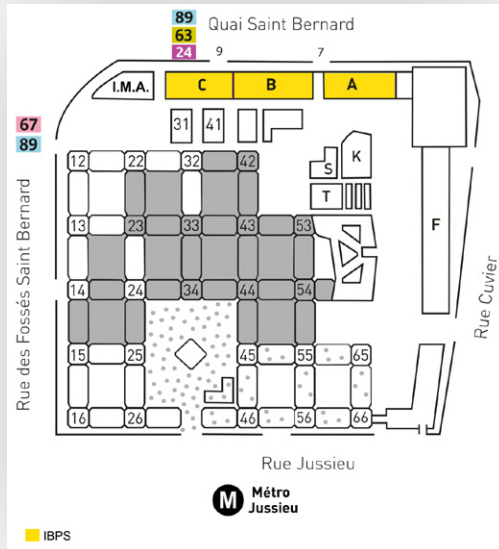
*Martin Sipkã and Michal Pavelka*

**Abstract.** We propose a novel approach for learning the evolution that employs differentiable neural networks to approximate the full GENERIC structure. Instead of manually choosing the fitted parameters, we learn the whole model together with the evolution equations. We can reconstruct the energy and entropy functions for the system under various assumptions and accurately capture systems behaviour for a double thermoelastic pendulum and a rigid body.

# Location Information



Comment venir à scai ?  
 Longer l'allée à droite de l'entrée principale jusqu'au dernier bâtiment « Esclangon ».  
 A l'intérieur du bâtiment Esclangon, prenez la 1<sup>re</sup> sortie à gauche puis l'escalier à droite. scai se situe en haut des marches.



## | List of restaurants



### Looking for a restaurant nearby La Sorbonne or Jussieu site?

The area is full of nice restaurants with French, Italian, American, Chinese specialities.

Scan the QR code below to get the list:



## | WiFi Access



**July 21<sup>st</sup>**  
Sorbonne site



ID: **GSI\_21**  
Password: **6ks896si**

---

**July 22<sup>nd</sup> & 23<sup>rd</sup>**  
Jussieu site



**A personal code for each attendee will be available  
at the Registration Desk.**

Feel free to ask for one!

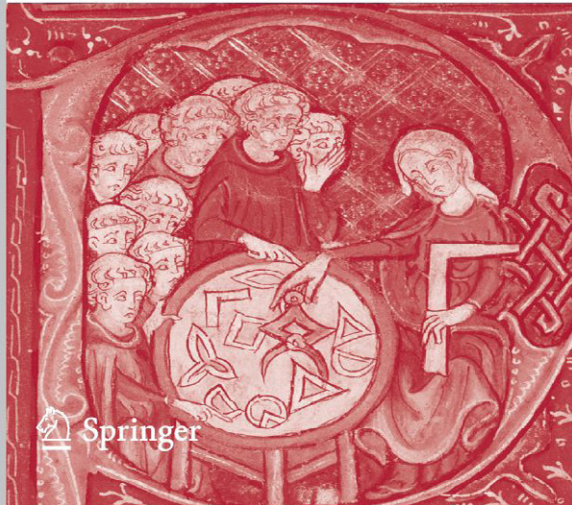
# | GSI'21 Proceedings

Frank Nielsen  
Frédéric Barbaresco (Eds.)

LNCS 12829

## Geometric Science of Information

5th International Conference, GSI 2021  
Paris, France, July 21–23, 2021  
Proceedings



<https://www.springer.com/gp/book/9783030802080>

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# Computational Geometric Science of Information: GEOMSTATS HACKATHON

If you are interested in:

- ✓ using and understanding existing implementations of differential geometry,
- ✓ implementing ideas, examples, experiments for a/your research paper,
- ✓ adding hands-on exercises or examples to your differential geometry classes,
  - ✓ learning how to code differential geometric structures,
  - ✓ learning how to contribute to an open-source GitHub project,

**Come to the hackathon!**

**When?**

21<sup>st</sup>, 22<sup>nd</sup> & 23<sup>rd</sup> of July from 12.30 pm to 2.30 pm (lunch break)

**Where?**

Esclançon building at SCAI, 2<sup>nd</sup> floor / (see map 44)

**How?**

Metro: Jussieu (lines 7 and 10)

**Nicolas Guigui** and **Nina Miolane** will be available to answer your questions about computational geometric science of information, guide you through existing implementations, and help you translate your ideas into code. Feel free to join any day, for any duration, with or without a computational project in mind, with or without coding experience!



## Welcome Cocktail

**The Welcome Cocktail Reception will take place on  
July 21<sup>st</sup> at 8:00 pm at the Jussieu Site.**

Follow Signs to the SCAI building (also see map page 44).

12-minute walk from Richelieu site or you can take the metro line 10 from Cluny La Sorbonne straight to Jussieu.

We look forward to seeing everyone there!



## Gala Dinner

**The Gala Dinner will be a boat cruise on the Seine and will take place on  
July 22<sup>nd</sup> at 8:00 pm.**

Anyone attending the Gala Dinner should plan to meet at Port d'Arsenal to get on the boat (see map page 1.2). If you are heading straight from the Jussieu site, it is a 19 minute walk across the Seine via Pont de Sully, or you can take metro line 7 from Jussieu to Sully-Morland and walk from there.

The gala dinner is included in the full registrations only.